

Answers to Coin Toss 2 of 5

Guided Activity 1: 2 out of 5

Do the experiment once (Click **Toss**). Do you understand what a winning combination is?

Answer: You must have 2 or more Heads on the board.

Do the experiment 1000 times (Click **1000**).

- What is the percent of **Wins**? Answer: ~66%
- What is the percent of **Losses**? Answer: 1-Above = ~34%
- Here, which should be easier to calculate - the theoretical probability of winning or losing? Answer: Losing, since % is smaller.

Now for some dreaded theory ...

1. An example of a winning combination is TTHHT. What is the probability of getting this combination? Answer: $0.6 \cdot 0.6 \cdot 0.4 \cdot 0.4 \cdot 0.6 = 0.035$
2. Another example of a winning combination is HHTHT. What is the probability of getting this combination? Answer: $0.4 \cdot 0.4 \cdot 0.6 \cdot 0.4 \cdot 0.6 = 0.023$
3. A third example is HTTHH. This combination is "similar to" example 1 or example 2? Answer: Similar to example 2 since it has 3H and 2T so same probability.

- What "identifies" the combinations with probability: $(0.4)^4 \cdot (0.6)^1$? Answer: They have 4H and 1T.
- How many different combinations are there of this type (i.e. with this probability)?

Think combinatorics. Answer: $C_{5,4} = \binom{5}{4} = \frac{5!}{4!(5-4)!} = 5$ (T can be in 5 places.)

Losing combinations

- How many **types** of losing combinations are there? Answer: Combinations with 0 Heads and combinations with 1 head.
- What is the **probability of each of these types** of losing combinations?

Answer: 0 Heads – 1 combination TTTTT or $C_{5,0} = \binom{5}{0} = \frac{5!}{0!(5-0)!} = 1$ with probability $p(0 \text{ Heads}) = (0.6)^5 = 0.078$ and

1 Head – 5 combinations HTTTT, THTTT, ... $C_{5,1} = \binom{5}{1} = \frac{5!}{1!(5-1)!} = 5$ with probability $p(1 \text{ Heads}) = (0.6)^4(0.4) = 0.052$ and

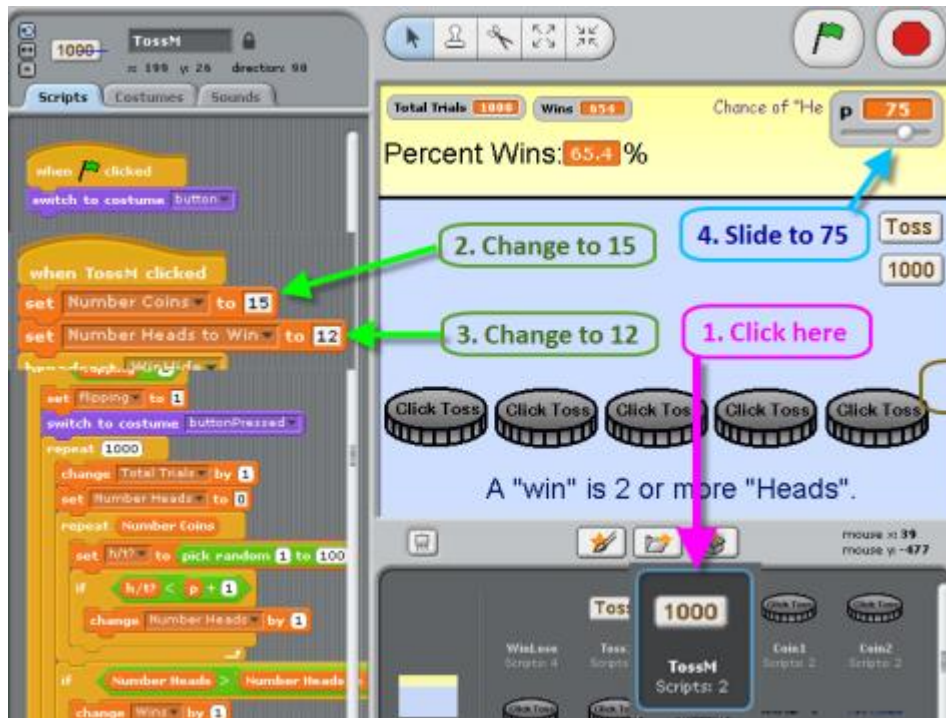
- What is the probability of losing? Answer: $\text{Pr}(\text{Losing}) = 1 \cdot 0.078 + 5 \cdot 0.052 = 0.338$

Winning combinations

- What is the probability of winning? Answer: $\text{Pr}(\text{Winning}) = 1 - 0.338 = 0.662$
- Does this correspond to your empirical results when you ran the experiment 1000 times? Answer: Hopefully 😊

Guided Activity 2: Other Bernoulli* experiments

Example: What is the probability of 12 or more successes in 15 trials of a Bernoulli experiment with $p=0.75$?



- Calculate the theoretical probability of winning.

Recall: The theoretical probability of getting exactly k successes in n trials with a probability p of success in any individual trial

$$\Pr(k, n, p) = \binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

Think before calculating: What are the possible values for k ?

Answer: 0, 1, 2, ..., n (Don't forget 0 successes when calculating complements!)

$\Pr(\text{Winning}) =$

$$\begin{aligned} & \Pr(12, 15, 0.75) + \Pr(13, 15, 0.75) + \Pr(14, 15, 0.75) + \Pr(15, 15, 0.75) = \\ & = \frac{15!}{12!(3)!} \cdot 0.75^{12} \cdot 0.25^3 + \frac{15!}{13!2!} \cdot 0.75^{13} \cdot 0.25^2 + \frac{15!}{14!1!} \cdot 0.75^{14} \cdot 0.25^1 + \frac{15!}{15!0!} \cdot 0.75^{15} \cdot 0.25^0 = \\ & = 0.46 = 46\% \end{aligned}$$

- Does your theory match your experiment? Answer: Hopefully 😊