Answers to Coin Toss 2 of 5

Guided Activity 1: 2 out of 5

Do the experiment once (Click Toss). Do you understand what a winning combination is?
Answer: You must have 2 or more Heads on the board.

Do the experiment 1000 times (Click 1000).

- What is the percent of Wins? Answer: ~66%
- What is the percent of Losses? Answer: 1-Above = ~34%
- Here, which should be easier to calculate - the theoretical probability of winning or losing? Answer: Losing, since % is smaller.
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Now for some dreaded theory ...

1. An example of a winning combination is TTHHT. What is the probability of getting this combination? Answer: 0.6*0.6*0.4*0.4*0.6=0.035
2. Another example of a winning combination is HHTHT. What is the probability of getting this combination? Answer: 0.4*0.4*0.6*0.4*0.6=0.023
3. A third example is HTTHH. This combination is "similar to" example 1 or example 2?
   Answer: Similar to example 2 since it has 3H and 2T so same probability.

- What "identifies" the combinations with probability: $(0.4)^4 \cdot (0.6)^1$?
  Answer: They have 4H and 1T.
- How many different combinations are there of this type (i.e. with this probability)?
  Think combinatorics. Answer: $C_{5,4} = \binom{5}{4} = \frac{5!}{4!(5-4)!} = 5$ (T can be in 5 places.)

Losing combinations

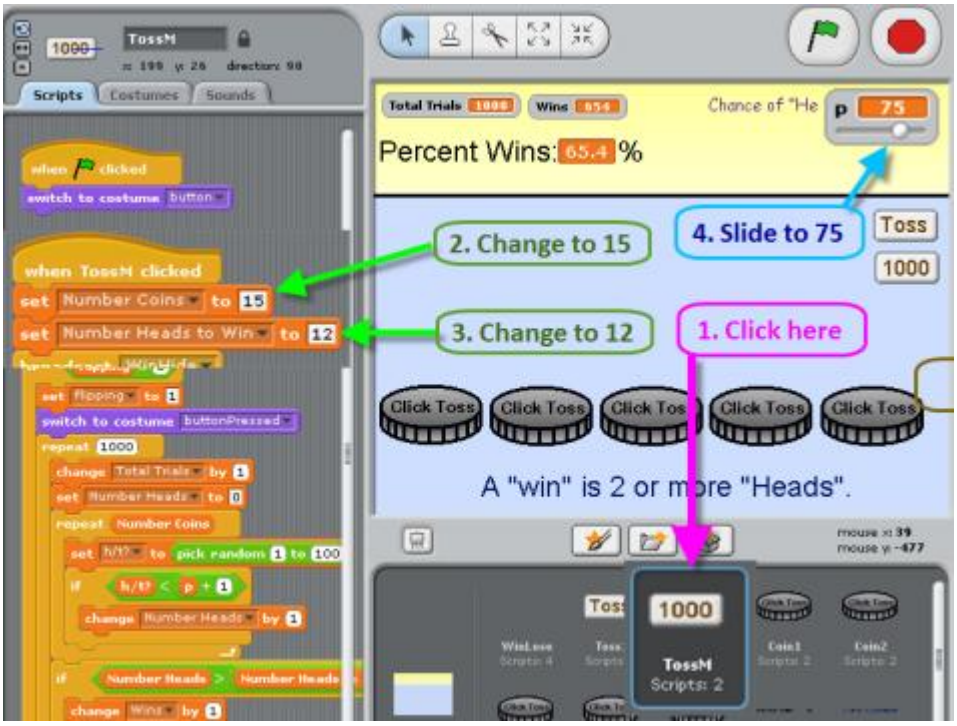
- How many **types** of losing combinations are there?
  Answer: Combinations with 0 Heads and combinations with 1 head.
- What is the **probability of each of these types** of losing combinations?
  Answer: 0 Heads – 1 combination TTTTT or $C_{5,0} = \binom{5}{0} = \frac{5!}{0!(5-0)!} = 1$ with probability $p(0 \text{ Heads}) = (0.6)^5 = 0.078$ and
  1 Head – 5 combinations HTTTT, THTTT,... $C_{5,1} = \binom{5}{1} = \frac{5!}{1!(5-1)!} = 5$ with probability $p(1 \text{ Heads}) = (0.6)^4(0.4) = 0.052$ and
- What is the probability of losing? Answer: Pr(Losing) = 1*0.078 + 5*0.052 = 0.338

Winning combinations

- What is the probability of winning? Answer: Pr(Winning) = 1-0.338 = 0.662
- Does this correspond to your empirical results when you ran the experiment 1000 times? Answer: Hopefully ☺

Guided Activity 2: Other Bernoulli* experiments

Example: What is the probability of 12 or more successes in 15 trials of a Bernoulli experiment with p=0.75 ?



- Calculate the theoretical probability of winning.

  Recall: The theoretical probability of getting exactly k successes in n trials with a probability p of success in any individual trial

$$\Pr(k,n,p)=\binom{n}{k}p^k(1-p)^{n-k}=\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

  Think before calculating: What are the possible values for k?
  Answer: 0, 1, 2, ….,n (Don't forget 0 successes when calculating complements!)

  Pr(Winning)=

$$\Pr(12,15,0.75)+\Pr(13,15,0.75)+\Pr(14,15,0.75)+\Pr(15,15,0.75)=$$

$$=\frac{15!}{12!(3)!}0.75^{12}\cdot 0.25^3+\frac{15!}{13!2!}0.75^{13}\cdot 0.25^2+\frac{15!}{14!1!}0.75^{14}\cdot 0.25^1+\frac{15!}{15!0!}0.75^{15}\cdot 0.25^0=$$

$$=0.46=46\%$$

- Does your theory match your experiment? Answer: Hopefully ☺