GeoGebra – Boat Landing 1

Level: 8^{th} - 10^{th} grade

Using GeoGebra to make a simulation

Author: Linda Fahlberg-Stojanovska

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Produced with: Camtasia Studio

www.mathcasts.org & math247.pbwiki.com

Key Concepts from GeoGebra

1. Draw points , line segments , polygons .
2. Make parameters variable using slider  .
3. Using the Input field.
   a. Input points.
   b. Input functions.
   c. Using the function: Function[] to restrict domain.
4. Run a simulation by animating a slider with arrow keys.

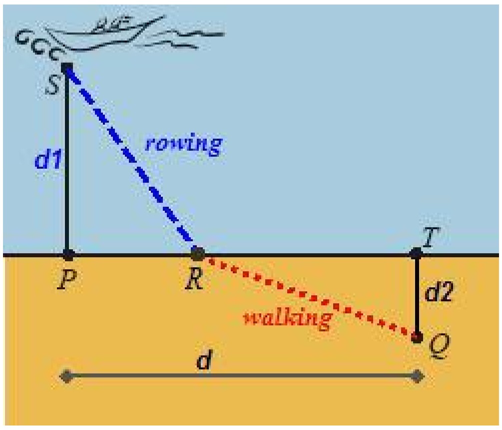
Key Concepts from Mathematics – Student should understand:

1. Pythagoras' theorem for a right-triangle: $a^2 + b^2 = c^2$.
2. The formula for distance, speed (velocity) and time
   $d = st$ or $s = v \cdot t$ (iso version)
3. Finding the minimum of a function from its graph.

Boat Landing Problem

Problem setting: A man with a boat at point S at sea wants to get to point Q inland. Point S is distance d1 from the closest point P on the shore, point Q is distance d2 from the closest point T on the shore. The points P and T are at a distance of d from each other.

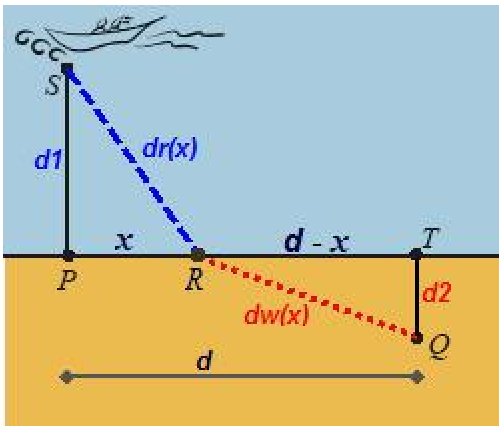
Question: If the man rows with a speed of v_r and walks with a speed of v_w at what point R should he beach the boat in order to get from point S to point Q in the least possible time?



The question asks us to find **the point *R*** which **minimizes** travel **time**.

We will do a *standard* but "mathematically bad" thing and let ***R*** be both a moving point along the shore and **the result** (when we find it).

Let ***x*** be the length of $\overline{PR}$. Then ***d-x*** is the length of $\overline{RT}$.



$$dr(x) = \sqrt{d1^2 + x^2}$$

$$dw(x) = \sqrt{d2^2 + (d-x)^2}$$

$$tr(x) = \frac{dr(x)}{v_r}$$

$$tw(x) = \frac{dw(x)}{v_w}$$

$$t(x) = tr(x) + tw(x)$$

The *specific* **x-value** at which the **Time function** $t(x)$ **has a minimum** gives us **the point *R*** asked for in the question.

The idea of a **Simulator** is to be able to run an experiment many times with **different input values** and see what changes.

What are our input variables: d d1 d2 v_r v_w

Sample input values:

Exp.1: d=8km d1=5km d2=2km $v_r = 7\,\text{km/h}$ $v_w = 3\,\text{km/h}$

Exp.2: d=5km d1=3km d2=0km $v_r = 2\,\text{km/h}$ $v_w = 10\,\text{km/h}$

Use GeoGebra - Sliders

Script-o-matic

1. Start GeoGebra.
2. Open the starter file: boat_sim1_starter.ggb (not required).

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3. Make Sliders - Input Slider Values

slider variable	low (min)	high (max)	increment
d	**1**	**11**	**1**
d1	**1**	**6**	**1**
d2	**0**	**5**	**1**
vr	**1**	**11**	**1**
vw	**1**	**11**	**1**

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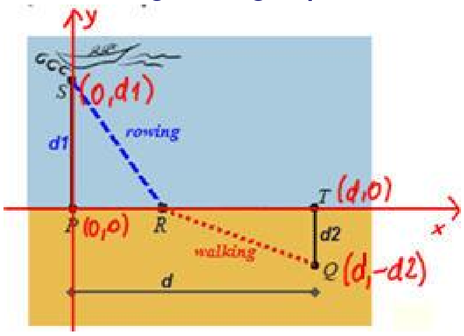
4. Set Sample Experiment Values – Move Slider Values

Specific problem setting: A man with a boat at point S at sea wants to get to point Q inland. Point S is distance d1=4km from the closest point P on the shore, point Q is distance d2=2km from the closest point T on the shore and point P and T are at a distance of d=8km.

Question: If the man rows with a speed of $v_r = 9\,\text{km/h}$ and walks with a speed of $v_w = 2\,\text{km/h}$ at what point R should he beach the boat in order to get from point S to point Q in the least possible time?

5. Moving the graph into GeoGebra – Inputting the Points



Input Point
P=(0,0)
S=(0,d1)
T=(d,0)
Q=(d,-d2)

6. Input the Functions

$$dr(x)=\sqrt{d1^2+x^2}$$

$$dw(x)=\sqrt{d2^2+(d-x)^2}$$

$$tr(x)=\frac{dr(x)}{v_r}$$

$$tw(x)=\frac{dw(x)}{v_w}$$

$$t(x)=tr(x)+tw(x)$$

Input function
dr(x)=sqrt(d1^2+x^2)
dw(x)=sqrt(d2^2+(d-x)^2)
tr(x)=dr(x)/vr
tr(x)=dr(x)/vr
t(x)=tr(x)+tw(x)

7. Restrict the function t(x) to [0,d]

Input: time(x)=Function[t(x),0,d]

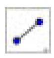
8. Make the simulator

a. Make slider for r.

slider variable	low (min)	high (max)	increment
r	**1**	**11**	**1**

b. Make point R on x-axis and point Time on function t(x).

Input Point
R=(r,0)
TIME=(r,t(r))

c. Make rowing line segment and walking line segment 

Connect S to R and then R to Q.

Extras

10. Make text for simulator 

a. Make text for R: "R = "+r

b. Make text for Time: "Time = "+t(r)

11. Make water and land

a. Make fourth points for upper and lower rectangle S1 and Q1.

Input Point
S1=(d,d1)
Q1=(0,-d2)

b. Make polygons