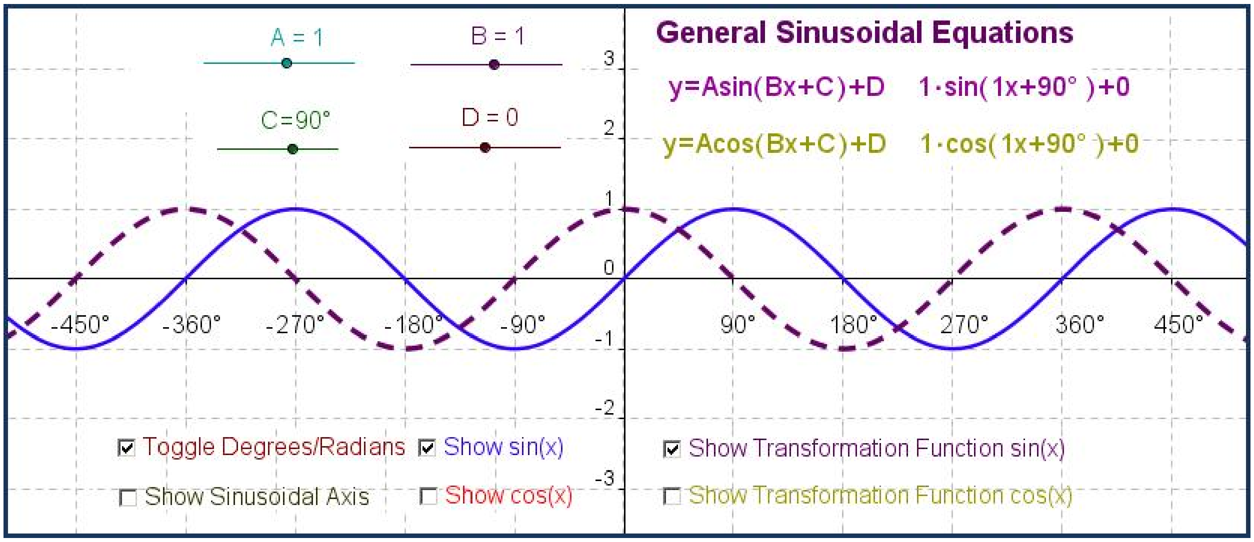
General Sinusoidal Functions

In this activity, we will explore transformations of the sine $y=\sin(x)$ and cosine $y=\cos(x)$ functions. These transformations look like this: $y=A\sin(Bx+C)+D$ and $y=A\cos(Bx+C)+D$.

InterActivity: Trig_Transformations



Take a few moments and explore the applet. Use the questions below to help guide your exploration. We want to see how the parameters A, B, C and D transform sine and cosine.

Before you begin, toggle the ☒ Toggle Degree/Radians Mode to Degrees.

Check that A=1, B=1, C=0° and D=0.

Check that ☒ Show sin(x) and ☒ Show Transformation Function sin(x) are the only boxes checked. You should see the solid blue graph $y=\sin(x)$ and a dashed purple graph $y=A\sin(Bx+C)+D$ (the transformed graph) "on top" of it.

Questions:

0. Why is the transformed graph on top of the graph of sin(x)?

Answer: With A=1, B=1, C=0° and D=0, the transformed function is:

$$y=A\sin(Bx+C)+D=1\sin(1x+0)+0=\sin(x)$$. That is, they are the same functions.

We work on D.

1. Move slider D in any direction. What transformation does this represent?

<u>Answer: Changing D moves the function up and down. It adds or subtracts from the y-value.</u>

2. Move D in the positive direction. What effect does moving D in the positive direction have on the transformation function $y = A\sin(Bx + C) + D$? What if D is negative?

Answer: Changing D in the positive direction moves the function up the value of D. Changing D in the negative direction moves the function down the value of D.

Set slider D=0. We work on A.

3. Adjust slider A to a positive and then to a negative value. What type of transformation does this represent?

Answer: Changing A in the positive direction increases the *vertical* size of the waves. Changing A in the negative direction flips the wave vertically and increases the size of the waves.

4. Set slider A=3. What happened to $y = A\sin(Bx + C) + D$? What is the amplitude (height of a wave) of this transformed function when A=3?

Answer: Changing A=3 increases the *vertical* size of the waves from 1 to 3 so that the amplitude of the transformed function $y = A\sin(Bx + C) + Dy = 3\sin(1x + 0) + 0 = 3\sin(x)$ is *amplitude*=3.

5. Set slider A=5. What is the amplitude of the transformed function?

Answer: Changing A=5 increases the *vertical* size of the waves from 1 to 5 so that the amplitude of the transformed function $y = 5\sin(1x + 0) + 0 = 5\sin(x)$ is *amplitude*=5.

6. Set the slider A to be negative. What happens to the transformed function? Set A= -4. What is the amplitude?

Answer: Changing A=-4 flips the wave vertically (around the x-axis) and increases the size of the waves from 1 to 4 so that the amplitude of the transformed function $y = -4\sin(1x + 0) + 0 = -4\sin(x)$ is *amplitude*=4.

7. What can we conclude about the value of the amplitude with respect to A?

Answer: Since height is always positive, amplitude is always positive so we need to use absolute value. The amplitude of the transformed function $y = A\sin(1x + 0) + 0 = A\sin(x)$ is *amplitude*=|A|=abs (A).

Set A=1. We work on B.

8. Move slider B to be a positive number other than 1. What transformation does this represent?

Answer: Changing B *shrinks* the function. It now has a *smaller period*.

9. Set B=2. What is the period of the transformed sine function when B=2? What if B=4?

Answer: Setting B=2 *shrinks* the function by two times so the transformed function

$y = 1\sin(2x+0)+0 = \sin(2x)$ has a period half the period of sin(x).

So the period of $y=\sin(2x)$ is: $period = \frac{2\pi}{2} = \pi$.

Answer: Setting B=4 *shrinks* the function by four times so the transformed function

$y = 1\sin(4x+0)+0 = \sin(4x)$ has a period one-fourth the period of sin(x).

So the period of $y=\sin(4x)$ is: $period = \frac{2\pi}{4} = \frac{\pi}{2}$.

10. What if B=0? What is the period of the transformed sine function?

Answer: Setting B=0 makes the transformed function $y = 1\sin(0x+0)+0 = 0$ or just $y=0$ (the x=axis) and this function has no period.

11. What if B is a negative number? Do you see the same results? Set B= 2 and then B= -2. What is the difference in the period of these 2 transformed sine functions?

Answer: Setting B to a negative number both shrinks the function and flips it horizontally (around the y-axis). There is no difference in the period of the functions when B= 2 and then B= -2.

The periods of: $y=\sin(2x)$ is: $period = \frac{2\pi}{|2|} = \pi$ and $y=\sin(-2x)$ is: $period = \frac{2\pi}{|-2|} = \pi$.

Set B=1. We work on C.

12. Finally, adjust slider C to a positive and then to a negative value. What transformation does this represent?

Answer: Changing C positively shifts the function horizontally (called a phase shift).

13. When C is positive, what direction does the transformed function move? When C is negative? Does this seem reasonable? Why?

Answer: Changing C positively shifts the function to the *left.* Changing C negatively shifts the function to the *right.*

This makes sense because we know that horizontal shift is always the answer to the question "What makes the argument 0?". Since A=1, B=1 and D=0, we have $y = \sin(x + C)$ So the argument is (x+C) and the answer to the question is x+C=0 or x=-C. So the horizontal shift is –C. If C is positive, this shift is to the left and if C is negative, this shift is to the right.

We have explored the sine function in great detail. We will now look at the cosine function and answer the same questions. But before we begin, change sliders A, B, C or D and watch what happens to the General Sinusoidal Equations of cosine.

14. Make 4 conjectures on what will happen to the cosine function when each of the sliders A, B, C or D is changed.

Conjecture 1: If A is changed, the amplitude will change. If A is negative, the function will do a vertical flip around the x-axis. (If |A|>1 the function will stretch vertically and if |A|<1 the function will shrink vertically.)

Conjecture 2: If B is changed, the period will change. If B is negative the function will do a horizontal flip around the y-axis. (If |B|>1 the function will shrink horizontally and if |B|<1 the function will stretch horizontally.)

Conjecture 3: If C is changed, the function will shift horizontally. If C>0 the function shift to the left and if C<0 the function will shift to the right.

Conjecture 4: If D is changed the function will shift vertically. If D>0 the function shift up and if D<0 the function will shift down.

Set A=1, B=1, C=0° and D=0

Check that ☒ Show cos(x) and ☒ Show Transformation Function cos(x) are the only boxes checked. You should see the solid red graph $y = \cos(x)$ and a dotted brown graph $y = A\cos(Bx + C) + D$ (the transformed graph) "on top" of it.

15. Explore the transformations on $y = \cos(x)$ by changing the sliders and looking at transformed graph $y = A\cos(Bx + C) + D$ and then do Q1-Q14 with cosine in place of sine.

Answers are the same as above!

16. Are there any differences between the transformations for sine and cosine?

Answer: No.

Conclusions – you should test each of your answers by varying all of A, B, C and D.

Easy ones (affect the y-coordinates)

- For $y = \sin(x)$, the *amplitude*=1.

  Answers: For $y = A\sin(Bx + C) + D$ the amplitude=|A|.

  For $y = \cos(x)$, the *amplitude*=1. For $y = A\cos(Bx + C) + D$ the amplitude=|A|.

- For $y = \sin(x)$, the *vertical shift*=0 and the sinusoidal axis is *y*=0.

  Answers: For $y = A\sin(Bx + C) + D$, the *vertical shift*=D and the sinusoidal axis is *y*=D.

  For $y = \cos(x)$, the *vertical shift*=0 and the sinusoidal axis is *y*=0.

  For $y = A\cos(Bx + C) + D$, the *vertical shift*=D and the sinusoidal axis is *y*=D.

Harder ones (affect the x-coordinates)

- For $y = \sin(x)$ the *period=2π.*

  Answers: For $y = A\sin(Bx + C) + D$ the *period* $= \frac{2\pi}{|B|}$.

  For $y = \cos(x)$, the *period=2π*. For $y = A\cos(Bx + C) + D$ the *period* $= \frac{2\pi}{|B|}$.

- The horizontal shift or phase shift of $y = \sin(x)$ is *phase shift*=0. Now let B=1.

  Answers: For $y = A\sin(Bx + C) + D$ the *phase shift* = –C.

  For $y = \cos(x)$, the *phase shift*=0. For $y = A\cos(Bx + C) + D$ the *phase shift* = –C.

Challenger (affects the x-coordinates)

See if you can determine the *phase shift* for $y = A\sin(Bx + C) + D$.

Hint: Fix C and change B. Write down the phase shifts. Then fix B and change C.

Answers: For $y = A\sin(Bx + C) + D$ the *phase shift* $= \frac{-C}{B}$.

> Thinking is: "What value of x makes the parenthetical expression of the function 0?" The parenthetical expression is $(Bx + C)$.
>
> We set this equal to 0: Bx+C=0 $\Rightarrow x = \frac{-C}{B}$. So the *phase shift* $= \frac{-C}{B}$.

For $y = \cos(x)$, the *phase shift*=0. For $y = A\cos(Bx + C) + D$, the *phase shift* $= \frac{-C}{B}$.

Definitions

Sinusoidal functions: Functions of the form $y = A\sin(Bx + C) + D$ and $y = A\cos(Bx + C) + D$ are called sinusoidal functions (or general sinusoidal functions). They are transformations of the sine and cosine function.

Examples: $y = \sin(x)$, $y = 3\cos(x - 90°) + 2$, $y = \cos(\frac{x}{3} + \pi)$, $y = \frac{1}{2} \cdot \sin(x - 1) - 3$.

Amplitude: The amplitude of a sinusoidal function is *amplitude=½(maximum-minimum)* . Amplitude is always positive.

Examples: For $y = \sin(x)$ the *amplitude* = 1 and for $y = \frac{1}{2}\cos(x) + 3$ the *amplitude* = 0.5.

Sinusoidal axis: The sinusoidal axis of a sinusoidal function is the *horizontal line halfway between the maximum and the minimum*.

Examples: For $y = \cos(x)$, the sinusoidal axis: y=0 (the x-axis) and

for $y = 7\cos(\frac{x}{3} + 2) + 1$, the sinusoidal axis: y=1.

Period: The period of a sinusoidal function $y = A\sin(Bx + C) + D$ is the smallest number ω such that $y(x + \omega) = y(x)$ for all real numbers x.

Examples: The period of $y = \sin(x)$ is ω=2π and the period of $y = 7\cos(\frac{x}{3} + 2) + 1$ is ω=6π.

Phase shift: The phase shift[1] of a sinusoidal function is the horizontal shift of the function from its base function. That is, the phase shift of $y = A\sin(Bx + C) + D$ is the amount of horizontal shift between this function and $y = \sin(x)$.

Examples: For $y = \cos(x)$ the *phase shift* = 0 and for $y = 7\sin(x - 90°)$ the *phase shift* = $\pi/2$

and for $y = 7\cos(\frac{x}{3} + 2) + 1$ the *phase shift* = -6.

Remember: Horizontal shift is always the answer to the question:
"What value of x makes the parenthetical expression of the function 0?"

So for $y = \cos(x)$, the expression is just (x) so x =0 makes the expression 0 and the *phase shift*=0.

For $y = 7\sin(x - 90°)$, the expression is (x-90°). We solve x-90°=0 to get *phase shift*=90°=π/2.

For $y = 7\cos(\frac{x}{3} + 2) + 1$, the expression is $(\frac{x}{3} + 2)$. We solve $\frac{x}{3} + 2 = 0$ to get *phase shift* = -6.

[1] The *phase* of a sinusoidal function is the number C.