General Sinusoidal Function

In this activity, we will explore transformations of the sine $y=\sin(x)$ and cosine $y=\cos(x)$ functions. These transformations look like this: $y=A\sin(Bx+C)+D$ and $y=A\cos(Bx+C)+D$.

Applet: Trig_Transformations.html

Take a few moments and explore the applet. Use the questions below to help guide your exploration. We want to see how the parameters A, B, C and D transform sine and cosine.

Before you begin, toggle the ☒ Toggle Degree/Radians Mode to Degrees. Check that A=1, B=1, C=0° and D=0. Check that ☒ Show sin(x) and ☒ Show Transformation Function sin(x) are the only boxes checked. You should see the solid blue graph $y=\sin(x)$ and a dashed purple graph $y=A\sin(Bx+C)+D$ (the transformed graph) “on top” of it.

Important: For each question substitute A, B, C and D and write out the transformed function!

Questions:

0. Why is the transformed graph on top of the graph of sin(x)?

We work on D.

1. Move slider D in any direction. What transformation does this represent?
2. Move D in the positive direction. What effect does moving D in the positive direction have on the transformation function $y=A\sin(Bx+C)+D$? What if D is negative?

Set slider D=0. We work on A.

3. Adjust slider A to a positive and then to a negative value. What type of transformation does this represent?
4. Set slider A=3. What happened to $y=A\sin(Bx+C)+D$? What is the amplitude (height of a wave) of this transformed function when A=3?
5. Set slider A=5. What is the amplitude of the transformed function?
6. Set the slider A to be negative. What happens to the transformed function? Set A= -4. What is the amplitude?
7. What can we conclude about the value of the amplitude with respect to A?

Set A=1. We work on B.

8. Move slider B to be a positive number other than 1. What transformation does this represent?
9. Set B=2. What is the period of the transformed sine function when B=2? What if B=4?
10. What if B=0? What is the period of the transformed sine function?
11. What if B is a negative number? Do you see the same results? Set B= 2 and then B= -2. What is the difference in the period of these 2 transformed sine functions?

Set B=1. We work on C.

12. Finally, adjust slider C to a positive and then to a negative value. What transformation does this represent? What transformation does this represent?
13. When C is positive, what direction does the transformed function move? When C is negative? Does this seem reasonable? Why?

We have explored the sine function in great detail. We will now look at the cosine function and answer the same questions. But before we begin, change sliders A, B, C or D and watch what happens to the General Sinusoidal Equations of cosine.

14. Make 4 conjectures on what will happen to the cosine function when each of the sliders A, B, C or D is changed.

Set A=1, B=1, C=0° and D=0

Check that ☒ Show cos(x) and ☒ Show Transformation Function cos(x) are the only boxes checked. You should see the solid red graph $y = \cos(x)$ and a dotted brown graph $y = A\cos(Bx + C) + D$ (the transformed graph) "on top" of it.

15. Explore the transformations on $y = \cos(x)$ by changing the sliders and looking at transformed graph $y = A\cos(Bx + C) + D$ and then do Q1-Q14 with cosine in place of sine.
16. Are there any differences between the transformations for sine and those for cosine?

Conclusions – you should test each of your answers by varying all of A, B, C and D.

Easy ones (affect the y-coordinates)

- For $y = \sin(x)$, the *amplitude*=1. What is the amplitude of $y = A\sin(Bx + C) + D$?

  What is the amplitude of $y = \cos(x)$ and the amplitude of $y = A\cos(Bx + C) + D$?
- For $y = \sin(x)$, the *vertical shift*=0 and the sinusoidal axis is *y*=0.

  What is the vertical shift of $y = A\sin(Bx + C) + D$? What is the sinusoidal axis?

  What are the vertical shifts and sinusoidal axes of $y = \cos(x)$ and of $y = A\cos(Bx + C) + D$?

Harder ones (affect the x-coordinates)

- For $y = \sin(x)$ the *period*=2*π.* What is the period of $y = A\sin(Bx + C) + D$?

  What is the period of $y = \cos(x)$ and the period of $y = A\cos(Bx + C) + D$?
- The horizontal shift or phase shift of $y = \sin(x)$ is *phase shift*=0. Now let B=1.

  What is the phase shift of $y = A\sin(x + C) + D$?

  What is the phase shift of $y = \cos(x)$ and of $y = A\cos(x + C) + D$?

Challenger (affects the x-coordinates)

See if you can determine the *phase shift* for $y = A\sin(Bx + C) + D$.

Hint: Fix C and change B. Write down the phase shifts. Then fix B and change C.

Definitions

Sinusoidal functions: Functions of the form $y = A\sin(Bx + C) + D$ and $y = A\cos(Bx + C) + D$ are called sinusoidal functions (or general sinusoidal functions). They are transformations of the sine and cosine function.

Examples: $y = \sin(x)$, $y = 3\cos(x - 90°) + 2$, $y = \cos(\frac{x}{3} + \pi)$, $y = \frac{1}{2} \cdot \sin(x - 1) - 3$.

Amplitude: The amplitude of a sinusoidal function is *amplitude=½(maximum-minimum)* . Amplitude is always positive.

Examples: For $y = \sin(x)$ the *amplitude* = 1 and for $y = \frac{1}{2}\cos(x) + 3$ the *amplitude* = 0.5.

Sinusoidal axis: The sinusoidal axis of a sinusoidal function is the *horizontal line halfway between the maximum and the minimum*.

Examples: For $y = \cos(x)$, the sinusoidal axis: y=0 (the x-axis) and

for $y = 7\cos(\frac{x}{3} + 2) + 1$, the sinusoidal axis: y=1.

Period: The period of a sinusoidal function $y = A\sin(Bx + C) + D$ is the smallest number ω such that $y(x + \omega) = y(x)$ for all real numbers x.

Examples: The period of $y = \sin(x)$ is ω=2π and the period of $y = 7\cos(\frac{x}{3} + 2) + 1$ is ω=6π.

Phase shift: The phase shift[1] of a sinusoidal function is the horizontal shift of the function from its base function. That is, the phase shift of $y = A\sin(Bx + C) + D$ is the amount of horizontal shift between this function and $y = \sin(x)$.

Examples: For $y = \cos(x)$ the *phase shift* = 0 and for $y = 7\sin(x - 90°)$ the *phase shift* = π/2

and for $y = 7\cos(\frac{x}{3} + 2) + 1$ the *phase shift* = -6.

Remember: Horizontal shift is always the answer to the question:
"What value of x makes the parenthetical expression of the function 0?"

So for $y = \cos(x)$, the expression is just (x) so x =0 makes the expression 0 and the *phase shift*=0.

For $y = 7\sin(x - 90°)$, the expression is (x-90°). We solve x-90°=0 to get *phase shift*=90°=π/2.

For $y = 7\cos(\frac{x}{3} + 2) + 1$, the expression is $(\frac{x}{3} + 2)$. We solve $\frac{x}{3} + 2 = 0$ to get *phase shift* = -6.

[1] The *phase* of a sinusoidal function is the number C.