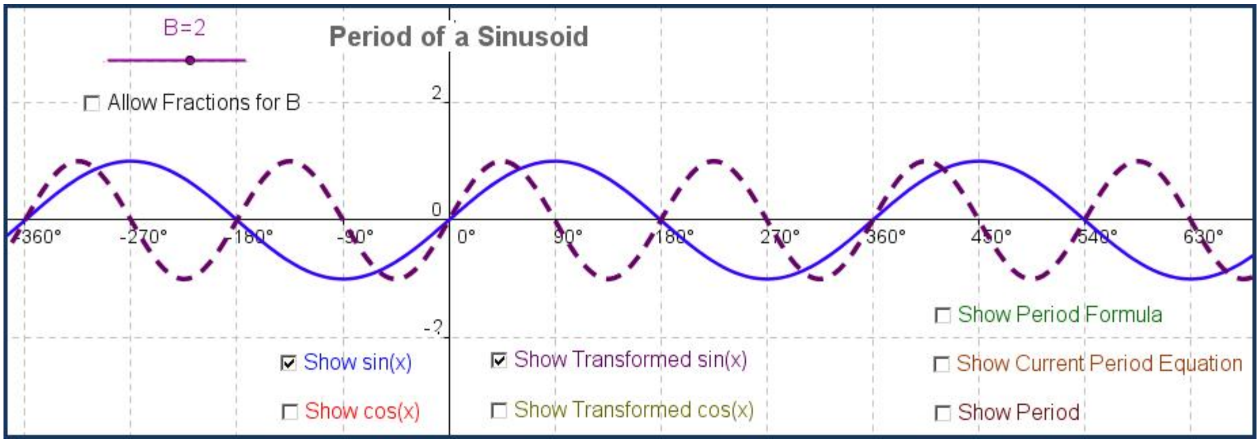
Period and Horizontal Dilation of Sine and Cosine Functions

In this activity, you will discover the relationship between the horizontal dilation B and the period P of: $y = \sin(Bx)$ and $y = \cos(Bx)$

Applet: Period_Horizontal_Dilation

Terminology: Horizontal dilation *stretches* or *compresses* the function along the x-axis.



Before you begin, check that: B=1 (and that the checkbox "Allow Fractions for B" is *unchecked*).

To avoid confusion, *when working on sine*, check Show sin(x) and Show Transformed sin(x) *and when working on cosine*, check Show cos(x) and Show Transformed cos(x).

Questions and Answers:

1. Write down the parent functions of y = sin(Bx) and y = cos(Bx). What is the value of B and what is the period P of the parent functions?

Answer: Parent functions are y = sin(x) and to y = cos(x), respectively. In both parent functions, the value of B is 1 and the period is P=2π (P=360°).

2. Move slider B in any direction (B≠0). What type of transformation does this represent?

Answer: Moving B in any direction gives a horizontal dilation or (since |B|>1) the function compresses horizontally.

3. What happens to y = sin(Bx) and to y = cos(Bx) when B increases in the positive direction?

Answer: When B increases in the positive direction and since B>1, the function compresses more and more.

4. Set B = 2. Write the equations for both the sine and cosine functions below.
Answer: Functions are $y = \sin(2x)$ and to $y = \cos(2x)$.

5. What is the period P of both sine and cosine under this horizontal dilation?
Answer: The period of the functions $y = \sin(2x)$ and to $y = \cos(2x)$ is $P=\pi$ ($P=180°$).

6. Set B = 4. Write the equations for both the sine and cosine functions below.
Answer: Functions are $y = \sin(4x)$ and $y = \cos(4x)$.

7. What is the period of both sine and cosine under this horizontal dilation?
Answer: The period of the functions $y = \sin(4x)$ and $y = \cos(4x)$ is $P = \frac{\pi}{2}$ ($P=90°$).

There seems to be a relationship between the period of a sinusoidal function and B. Look at Table 1. Let's see if you can see the pattern (Hint: Think about division).

Table 1: Positive Whole Numbers for B.

Dilation Factor	B = 1	B = 2	B = 4
Period of	$P = 2\pi$	$P = \pi$	$P = \pi/2$

8. Write an equation for the period P in terms of B when B>0.
Answer: The period of the functions $y = \sin(Bx)$ and to $y = \cos(Bx)$ is $P = \frac{2\pi}{B}$.

Note: Because we are not looking at B<0, at this time they will not write the correct equation.

9. Review question: Set B=1 and then B=-1. What changes in the graph of the sine function? What changes in the cosine function? Complete the equations: sin(-x)=? and cos(-x)=?

Answer: Correct answers for the sine function are: (a) The graphs of $y = \sin(x)$ and $y = \sin(-x)$ are reflections across the x-axis *or* (b) The graphs of $y = \sin(x)$ and $y = \sin(-x)$ are reflections across the y-axis. Correct answer for the cosine function are (a) The graphs of $y = \cos(x)$ and $y = \cos(-x)$ are the same. $\sin(-x)=-\sin(x)$ (odd function) and $\cos(-x)=\cos(x)$ (even function).

10. Let's see what happens to the period when B is negative? Set B = –1. What is the period P of the sine and cosine function? What is the period P of the sine and cosine function when B = –2 and when B = –4? Fill in row2 of Table 2 below.
Answer: The period of $y = \sin(-x)$ is $P=2\pi$ (the function is not dilated, just inverted). The period of $y = \cos(-x)$ is $P=2\pi$ (the function is not dilated, nor inverted). The periods of $y = \sin(-2x)$ and $y = \cos(-2x)$ is $P=\pi$ and the periods of $y = \sin(-4x)$ and $y = \cos(-4x)$ is $P=\pi/2$.

11. Does the period change between when B is positive and B is negative?
Answer: No.

Table 2: Positive and Negative Whole Numbers for B.

Dilation Factor	B = 1	B = 2	B = 4	B = -1	B = -2	B = -4
Period	P = 2π	P = π	P = π /2	P = 2π	P = π	P = π /2
Period using Equation	$P=\frac{2\pi}{\lvert 1\rvert}=2\pi$	$P=\frac{2\pi}{\lvert 2\rvert}=\pi$	$P=\frac{2\pi}{\lvert 4\rvert}=\frac{\pi}{2}$	$P=\frac{2\pi}{\lvert -1\rvert}=2\pi$	$P=\frac{2\pi}{\lvert -2\rvert}=\pi$	$P=\frac{2\pi}{\lvert -4\rvert}=\frac{\pi}{2}$

12. Modify your equation for the period P from Question 8 to include both positive and negative values for B. You will have to change your expression for B. Use the table above to check whether your equation works.

Answer: Since period is always positive, we need to use absolute value: $P=\frac{2\pi}{|B|}$

Challenger questions:

Notice that slider B allows only whole numbers. Let us see if our equation for P from Question 11 works when B is a fraction.

First, let's see what happens to the functions when B is a fraction between 0 and 1.

- Click on the checkbox to toggle *Allow Fraction Values for B*

13. Move slider B from B= 1 to B= ½. What type of transformation does this represent?

Answer: Moving B from B= 1 to B= ½ stretches the function horizontally (a horizontal dilation).

14. Set B= ½. Write the equation for both the sine and cosine function below.

Answer: $y=\sin(\frac{x}{2})$ and $y=\cos(\frac{x}{2})$

15. What is the period of both sine and cosine under this horizontal dilation?

Answer: The period of these functions is P=4π.

16. Set B= ¼. Write the equation for both the sine and cosine function below and then find the period of these functions. Fill in row2 of Table 3 below.

Answer: $y=\sin(\frac{x}{4})$ and $y=\cos(\frac{x}{4})$ and the period of these functions is P=8π.

17. Now using your equation for P from Question 11, calculate the period P for B= ½ and B= ¼. Fill in row3 of Table 3 and check that row2 and row3 are the same!

Table 3: Positive Fractions for B

Dilation Factor	B = 1	B = ½	B = ¼
Period	P = 2π	P = 4π	P = 8π
Period using Equation	$P=\frac{2\pi}{\lvert 1\rvert}$ $=2\pi$	$P=\frac{2\pi}{\lvert\frac{1}{2}\rvert}=2\pi\cdot\frac{2}{1}$ $=2\pi\cdot 2=4\pi$	$P=\frac{2\pi}{\lvert\frac{1}{4}\rvert}=2\pi\cdot\frac{4}{1}$ $=2\pi\cdot 4=8\pi$

18. Let's look at B between -1 and 0. Complete Table 4 with B=-1, B=-½ and B=-¼. Check that row2 and row3 are the same.

Table 4: Negative Fractions for B

Dilation Factor	B = -1	B = -½	B = -¼
Period	P = 2π	P =	P =
Period using Equation	$P=\frac{2\pi}{\lvert -1\rvert}$ $=2\pi$	$P=\frac{2\pi}{\lvert\frac{-1}{2}\rvert}=2\pi\cdot\frac{2}{1}$ $=2\pi\cdot 2=4\pi$	$P=\frac{2\pi}{\lvert\frac{-1}{4}\rvert}=2\pi\cdot\frac{4}{1}$ $=2\pi\cdot 4=8\pi$

Conclusions (cross out incorrect responses in blocks and fill in blanks):

1. Changing the value of B dilates (stretches or compresses) the functions y = sin(Bx) and y = cos(Bx) along the [horizontal] [~~vertical~~] axis.
2. When |B|>1, the functions y = sin(Bx) and y = cos(Bx) are [~~stretched~~] [compressed] and the period P of these functions is [~~P > 2π~~] [P < 2π].
3. When the period P > 2**π** of the functions y = sin(Bx) and y = cos(Bx), this means that [~~|B|>1~~] [0<|B|<1] and the functions are [stretched] [~~compressed~~] horizontally.
4. The period of y = sin(Bx) and y = cos(Bx) is determined by the formula $P=\frac{2\pi}{\lvert B\rvert}$