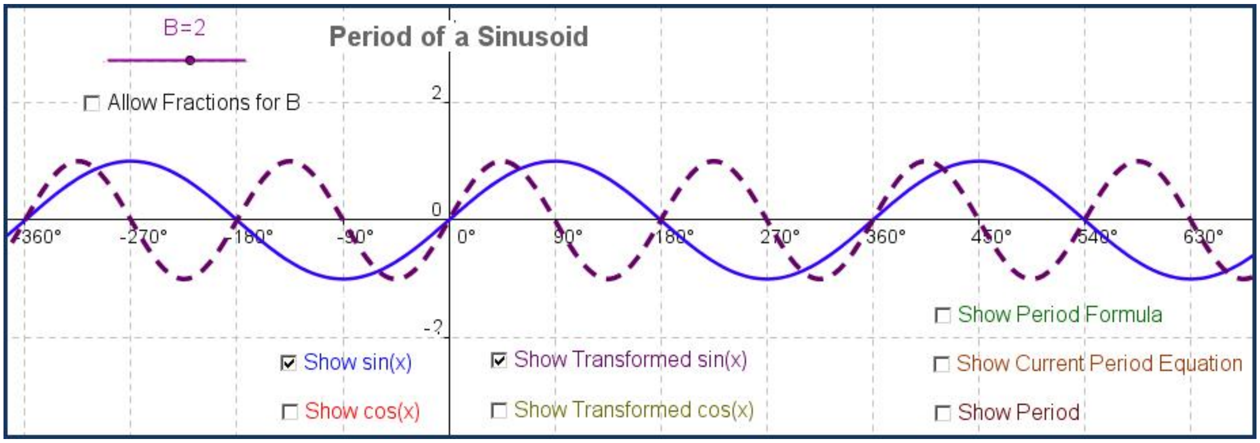
Period and Horizontal Dilation of Sine and Cosine Functions

In this activity, you will discover the relationship between the horizontal dilation B and the period P of: $y = \sin(Bx)$ and $y = \cos(Bx)$

Applet: Period_Horizontal_Dilation

Terminology: Horizontal dilation *stretches* or *compresses* the function along the x-axis.



Before you begin, check that: B=1 (and that the checkbox "Allow Fractions for B" is *unchecked*).

To avoid confusion, *when working on sine*, check Show sin(x) and Show Transformed sin(x) *and when working on cosine*, check Show cos(x) and Show Transformed cos(x).

Questions:

1. Write down the parent functions of $y = \sin(Bx)$ and $y = \cos(Bx)$. What is the value of B and what is the period of the parent functions?

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2. Move slider B in any direction (B≠0). What type of transformation does this represent?

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3. What happens to $y = \sin(Bx)$ and to $y = \cos(Bx)$ when B increases in the positive direction?

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4. Set B = 2. Write the equations for both the sine and cosine functions below.

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5. What is the period of both sine and cosine under this horizontal dilation?

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6. Set B = 4. Write the equations for both the sine and cosine functions below.

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7. What is the period of both sine and cosine under this horizontal dilation?

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There seems to be a relationship between the period of a sinusoidal function and B. Let's see if you can see the pattern (Hint: Think about division).

Table 1: Positive Whole Numbers for B.

Dilation Factor	B = 1	B = 2	B = 4
Period	P = 2π	P = π	P = π /2

8. Write an equation for the period P in terms of B when B>0.

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9. Review question: Set B=1 and then B=-1. What changes in the graph of the sine function? What changes in the cosine function? Complete the equations: sin(-x)=? and cos(-x)=?

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10. Let’s see what happens to the period when B is negative? Set B = –1. What is the period P of the sine and cosine function? What is the period P of the sine and cosine function when B = –2 and when B = –4? Fill in row2 of Table 2 below.

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11. Does the period change between when B is positive and B is negative?

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Table 2: Positive and Negative Whole Numbers for B.

Dilation Factor	B = 1	B = 2	B = 4	B = -1	B = -2	B = -4
Period	$P = 2\pi$	$P = \pi$	$P = \pi/2$			
Period using Equation						

12. Modify your equation for the period P from Question 8 to include both positive and negative values for B. You will have to change your expression for B. Use the table below to check whether your equation works.

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Challenger questions:

Notice that slider B allows only whole numbers. Let us see if our equation for P from Question 11 works when B is a fraction.

First, let's see what happens to the functions when B is a fraction between 0 and 1.

- Click on the checkbox to toggle *Allow Fraction Values for B*

13. Move slider B from B= 1 to B= ½. What type of transformation does this represent?

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14. Set B= ½. Write the equation for both the sine and cosine function below.

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15. What is the period of both sine and cosine under this horizontal dilation?

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16. Set B= ¼. Write the equation for both the sine and cosine function below and then find the period of these functions. Fill in row2 of Table 3 below.

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17. Now using your equation for P from Question 11, calculate the period P for B= ½ and B= ¼. Fill in row3 of Table 3 and check that row2 and row3 are the same!

Table 3: Positive Fractions for B

Dilation Factor	B = 1	B = ½	B = ¼
Period	P = 2π	P =	P =
Period using Equation			

18. Let's look at B between -1 and 0. Complete Table 4 with B=-1, B=-½ and B=-¼. Check that row2 and row3 are the same.

Table 4: Negative Fractions for B

Dilation Factor	B = -1	B = -½	B = -¼
Period	P = 2π	P =	P =
Period using Equation			

Conclusions (cross out incorrect responses in blocks and fill in blanks):

1. Changing the value of B dilates (stretches or compresses) the functions $y = \sin(Bx)$ and $y = \cos(Bx)$ along the [horizontal] [vertical] axis.
2. When $|B|>1$, the functions $y = \sin(Bx)$ and $y = \cos(Bx)$ are [stretched] [compressed] and the period P of these functions is [$P > 2\pi$] [$P < 2\pi$].
3. When the period $P > 2\pi$ of the functions $y = \sin(Bx)$ and $y = \cos(Bx)$, this means that [$|B|>1$] [$0<|B|<1$] and the functions are [stretched] [compressed] horizontally.
4. The period of $y = \sin(Bx)$ and $y = \cos(Bx)$ is determined by the formula [P=]