Period and Horizontal Dilation of Sine and Cosine Functions

In this activity, you will discover the relationship between the horizontal dilation B and the period P of: $y = \sin(Bx)$ and $y = \cos(Bx)$

Applet: Period_Horizontal_Dilation

Terminology: Horizontal dilation *stretches* or *compresses* the function along the x-axis.

Check that: B=1 (and that the checkbox "Allow Fractions for B" is *unchecked*).

1. Write down the parent functions of $y = \sin(Bx)$ and $y = \cos(Bx)$. What is the value of B and what is the period P of the parent functions?
2. Move slider B in any direction (B≠0). What type of transformation does this represent?
3. What happens to y = sin(Bx) and to y = cos(Bx) when B increases in the positive direction?
4. Set B = 2. Write the equations for both the sine and cosine functions below.
5. What is the period P of both sine and cosine under this horizontal dilation?
6. Set B = 4. Write the equations for both the sine and cosine functions below.
7. What is the period P of both sine and cosine under this horizontal dilation?

There seems to be a relationship between the period of a sinusoidal function and B.

Let's see if you can see the pattern (Hint: Think about division).

Dilation Factor	B = 1	B = 2	B = 4
Period	P = 2π	P = π	P = π /2

8. Write an equation for the period P in terms of B when B>0.
9. Review question: Set B=1 and then B=-1. What changes in the graph of the sine function? What changes in the cosine function? Complete the equations: sin(-x)=? and cos(-x)=?
10. Let's see what happens to the period when B is negative? Set B = –1. What is the period P of the sine and cosine function? What is the period P of the sine and cosine function when B = –2 and when B = –4? Fill in row2 of the table below.
11. Does the period change between when B is positive and B is negative?
12. Modify your equation for the period P from Question 8 to include both positive and negative values for B. You will have to change your expression for B. Use the table below to check whether your equation works.

Dilation Factor	B = 1	B = 2	B = 4	B = -1	B = -2	B = -4
Period	P = 2π	P = π	P = π /2			
Period using Equation						

Challenger questions:

Notice that slider B allows only whole numbers. Let us see if our equation for P from Question 11 works when B is a fraction.

First, let's see what happens to the functions when B is a fraction between 0 and 1.

- Click on the checkbox to toggle *Allow Fraction Values for B*

13. Move slider B from B= 1 to B= ½. What type of transformation does this represent?
14. Set B= ½. Write the equation for both the sine and cosine function below.
15. What is the period of both sine and cosine under this horizontal dilation?
16. Set B= ¼. Write the equation for both the sine and cosine function below and then find the period of these functions. Fill in row2 of the table below.
17. Now using your equation for P from Question 11, calculate the period P for B= ½ and B= ¼. Fill in row3 of the table below and check that row2 and row3 are the same!

Dilation Factor	B = 1	B = ½	B = ¼
Period	P = 2π	P =	P =
Period using Equation			

18. Let's look at B between -1 and 0. Make another table with B=-1, B=-½ and B=-¼. Check that row2 and row3 are the same.

Dilation Factor	B = -1	B = -½	B = -¼
Period	P = 2π	P =	P =
Period using Equation			

Conclusions (cross out incorrect responses in blocks and fill in blanks):

1. Changing the value of B dilates (stretches or compresses) the functions y = sin(Bx) and y = cos(Bx) along the [horizontal] [vertical] axis.
2. When |B|>1, the functions y = sin(Bx) and y = cos(Bx) are [stretched] [compressed] and the period P of these functions is [P > 2π] [P < 2π].
3. When the period P > 2π of the functions y = sin(Bx) and y = cos(Bx), this means that [|B|>1] [0<|B|<1] and the functions are [stretched] [compressed] horizontally.
4. The period of y = sin(Bx) and y = cos(Bx) is determined by the formula [P=]