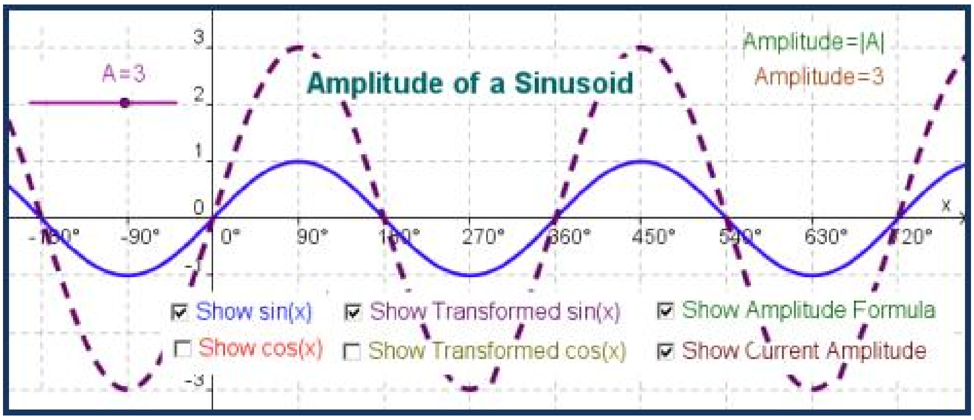
Amplitude and Vertical Dilation of Sine and Cosine Functions

In this activity, you will discover the relationship between the vertical dilation A and the amplitude of: $y = A\sin(x)$ and $y = A\cos(x)$

Applet: Amplitude_Vertical_Dilation

Terminology: Vertical dilation *stretches* or *compresses* the function along the y-axis.



Before you begin, check that: A=1 and that the checkbox "Allow Fractions for A" is *unchecked*.

Questions and Answers:

1. Write down the parent functions of $y = A\sin(x)$ and $y = A\cos(x)$. What is the value of A and what is the amplitude of these parent functions?

Answer: Parent functions are y = sin(x) and to y = cos(x), respectively. In both parent functions, the value of A is 1 and the *amplitude*=1.

2. Move slider A in any direction (A≠0). What type of transformation does this represent?

Answer: Moving A (A≠0) in any direction gives a vertical dilation or (since |A|>1), the transformation is a vertical stretch.

3. What happens to y = Asin(x) and to y = Acos(x) when A increases in the positive direction?

Answer: When A increases in the positive direction and since A>1, the functions stretch vertically more and more.

4. Set A = 2. Write the equations for both the sine and cosine functions below.

Answer: The equations are y = 2sin(x) and y = 2cos(x).

5. What is the amplitude of both sine and cosine under this vertical dilation?

Answer: The amplitudes of y = 2sin(x) and y = 2cos(x) is *amplitude*=2.

6. Set A = 4. Write the equations for both the sine and cosine functions below.

Answer: The equations are y = 4sin(x) and y = 4cos(x).

7. What is the amplitude of both sine and cosine under this vertical dilation?

Answer: The amplitudes of y = 4sin(x) and y = 4cos(x) is *amplitude*=4.

There seems to be a relationship between the amplitude of a sinusoidal function and A. Let's see if you can see the pattern.

Table 1: Positive Whole Numbers for A.

Dilation Factor	A = 1	A = 2	A = 4
Amplitude	*amplitude* = 1	*amplitude* = 2	amplitude = 4

8. Write an equation for the *amplitude* in terms of A when A>0.

Answer: The amplitude of the functions y = Asin(x) and to y = Acos(x) is *amplitude*=A.

Note: Because we are not looking at A<0, at this time they will not write the correct equation.

9. Review question: Set A=1 and then A=-1. What changes in the graph of the sine function? What changes in the cosine function? Complete the equations: -sin(x)=? and -cos(x)=?

Answer: Correct answers for the sine function are: (a) The graphs of y = sin(x) and y = sin(-x) are reflections across the x-axis *or* (b) The graphs of y = sin(x) and y = sin(-x) are reflections across the y-axis. Correct answer for the cosine function are (a) The graphs of y = cos(x) and y = cos(-x) are the same. sin(-x)=-sin(x) (odd function) and cos(-x)=cos(x) (even function).

10. Let's see what happens when A is negative? Set A = –1. What is the amplitude of the sine and cosine function? What is the amplitude of the sine and cosine function when A = –2 and when A = –4? Fill in row2 of Table 2 below.

Answer: The amplitude of y = sin(-x) is *amplitude*=1 (the function is not dilated, just inverted). The amplitude of y = cos(-x) is *amplitude*=1 (the function is not dilated, nor inverted). The amplitudes of y = sin(-2x) and y = cos(-2x) is *amplitude*=2 and the amplitudes of y = sin(-4x) and y = cos(-4x) is *amplitude*=4.

11. Does the *amplitude* change between when A is positive and A is negative?

Answer: No.

Table 2: Positive and Negative Whole Numbers for A.

Dilation Factor	A = 1	A = 2	A = 4	A = -1	A = -2	A = -4
Amplitude	*ampl* = 1	*ampl*= 2	*ampl*= 4	*ampl* = 1	*ampl*= 2	*ampl*= 4
Amplitude using Equation	*ampl*=|A| =|1|=1	*ampl*=|A| =|2|=2	*ampl*=|A| =|4|=4	*ampl*=|A| =|-1|=1	*ampl*=|A| =|-2|=2	*ampl*=|A| =|-4|=4

12. Modify your equation for amplitude from Question 8 to include both positive and negative values for A. You will have to change your expression for A. Use Table 2 above to check whether your equation works.

Answer: Since amplitude is always positive, we need to use absolute value: *Amplitude* = |A|

Challenger questions:

Notice that slider A allows only whole numbers. Let us see if our equation for *amplitude* from Question 12 works when A is a fraction.

First, let's see what happens to the functions when A is a fraction between 0 and 1.

- Click on the checkbox to toggle *Allow Fraction Values for A*

13. Move slider A from A= 1 to A= ½. What type of transformation does this represent?

Answer: Moving A from A= 1 to A= ½ stretches the function vertically (a vertical dilation).

14. Set A= ½. Write the equation for both the sine and cosine function below.

Answer: $y = \frac{1}{2}\sin(x)$ and $y = \frac{1}{2}\cos(x)$

15. What is the amplitude of both sine and cosine under this vertical dilation?

Answer: The amplitude of these functions is *amplitude* = $\frac{1}{4}$

16. Set A= ¼. Write the equation for both the sine and cosine function below and then find the amplitude of these functions. Fill in row2 of the table below.

Answer: $y = \frac{1}{4}\sin(x)$ and $y = \frac{1}{4}\cos(x)$ and the amplitude of these functions is *ampl* = $\frac{1}{4}$.

17. Now using your equation for P from Question 12, calculate the amplitude P for A= ½ and A= ¼. Fill in row3 of the table below and check that row2 and row3 are the same!

Table 3: Positive Fractions for A

Dilation Factor	A = 1	A = ½	A = ¼
Amplitude	*amplitude* = 1	*amplitude* = ½	*amplitude* = ¼
Amplitude using Equation	*amplitude* =|A| =|1|=1	*amplitude* =|A| =|½|=½	*amplitude* =|A| =|¼|=¼

18. Let's look at A between -1 and 0. Complete Table 4 with A=-1, A=-½ and A=-¼. Check that row2 and row3 are the same.

Table 4: Negative Fractions for A

Dilation Factor	A = -1	A = -½	A = -¼
Amplitude	*amplitude* = 1	*amplitude* = ½	*amplitude* = ¼
Amplitude using Equation	*amplitude* =|A| =|-1|=1	*amplitude* =|A| =|-½|=½	*amplitude* =|A| =|-¼|=¼

Conclusions (cross out incorrect responses in blocks and fill in blanks):

1. Changing the value of A dilates (stretches or compresses) the functions y = Asin(x) and y = Acos(x) along the ~~horizontal~~ vertical axis.
2. When |A|>1, the functions y = Asin(x) and y = Acos(x) are stretched ~~compressed~~ and the amplitude A of these functions is amplitude > 1 ~~amplitude < 1~~.
3. When the amplitude < 1 of the functions y = Asin(x) and y = Acos(x), this means that ~~|A|>1~~ 0<|A|<1 and the functions are ~~stretched~~ compressed vertically.
4. The amplitude of y = Asin(x) and y = Acos(x) is determined by the formula

Amplitude = |A|