Amplitude and Vertical Dilation of Sine and Cosine Functions

In this activity, you will discover the relationship between the vertical dilation $A$ and the amplitude of: $y = A\sin(x)$ and $y = A\cos(x)$

Applet: [Amplitude_Vertical_Dilation](#)

Terminology: Vertical dilation stretches or compresses the function along the y-axis.

Before you begin, check that: $A=1$ and that the checkbox "Allow Fractions for $A$" is unchecked.

Questions and Answers:

1. Write down the parent functions of $y = A\sin(x)$ and $y = A\cos(x)$. What is the value of $A$ and what is the amplitude of these parent functions?
   
   **Answer:** Parent functions are $y = \sin(x)$ and to $y = \cos(x)$, respectively. In both parent functions, the value of $A$ is 1 and the **amplitude**=1.

2. Move slider $A$ in any direction ($A\neq0$). What type of transformation does this represent?
   
   **Answer:** Moving $A$ ($A\neq0$) in any direction gives a vertical dilation or (since $|A|>1$), the transformation is a vertical stretch.

3. What happens to $y = A\sin(x)$ and to $y = A\cos(x)$ when $A$ increases in the positive direction?
   
   **Answer:** When $A$ increases in the positive direction and since $A>1$, the functions stretch vertically more and more.

4. Set $A = 2$. Write the equations for both the sine and cosine functions below.
   
   **Answer:** The equations are $y = 2\sin(x)$ and $y = 2\cos(x)$. 

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5. What is the amplitude of both sine and cosine under this vertical dilation?
Answer: The amplitudes of \( y = 2\sin(x) \) and \( y = 2\cos(x) \) is \( \text{amplitude}=2 \).

6. Set \( A = 4 \). Write the equations for both the sine and cosine functions below.
Answer: The equations are \( y = 4\sin(x) \) and \( y = 4\cos(x) \).

7. What is the amplitude of both sine and cosine under this vertical dilation?
Answer: The amplitudes of \( y = 4\sin(x) \) and \( y = 4\cos(x) \) is \( \text{amplitude}=4 \).

There seems to be a relationship between the amplitude of a sinusoidal function and \( A \).
Let’s see if you can see the pattern.

<table>
<thead>
<tr>
<th>Dilation Factor</th>
<th>( A = 1 )</th>
<th>( A = 2 )</th>
<th>( A = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>( \text{amplitude}=1 )</td>
<td>( \text{amplitude}=2 )</td>
<td>( \text{amplitude}=4 )</td>
</tr>
</tbody>
</table>

8. Write an equation for the amplitude in terms of \( A \) when \( A > 0 \).
Answer: The amplitude of the functions \( y = A\sin(x) \) and \( y = A\cos(x) \) is \( \text{amplitude}=A \).
Note: Because we are not looking at \( A < 0 \), at this time they will not write the correct equation.

9. Review question: Set \( A = 1 \) and then \( A = -1 \). What changes in the graph of the sine function?
What changes in the cosine function? Complete the equations: \( -\sin(x) = ? \) and \( -\cos(x) = ? \)
Answer: Correct answers for the sine function are: (a) The graphs of \( y = \sin(x) \) and \( y = \sin(-x) \) are reflections across the x-axis or (b) The graphs of \( y = \sin(x) \) and \( y = \sin(-x) \) are reflections across the y-axis. Correct answer for the cosine function are (a) The graphs of \( y = \cos(x) \) and \( y = \cos(-x) \) are the same. \( \sin(-x) = -\sin(x) \) (odd function) and \( \cos(-x) = \cos(x) \) (even function).

10. Let’s see what happens when \( A \) is negative? Set \( A = -1 \). What is the amplitude of the sine and cosine function? What is the amplitude of the sine and cosine function when \( A = -2 \) and when \( A = -4 \)? Fill in row 2 of Table 2 below.
Answer: The amplitude of \( y = \sin(-x) \) is \( \text{amplitude}=1 \) (the function is not dilated, just inverted). The amplitude of \( y = \cos(-x) \) is \( \text{amplitude}=1 \) (the function is not dilated, nor inverted). The amplitudes of \( y = \sin(-2x) \) and \( y = \cos(-2x) \) is \( \text{amplitude}=2 \) and the amplitudes of \( y = \sin(-4x) \) and \( y = \cos(-4x) \) is \( \text{amplitude}=4 \).

11. Does the amplitude change between when \( A \) is positive and \( A \) is negative?
Answer: No.
12. Modify your equation for amplitude from Question 8 to include both positive and negative values for A. You will have to change your expression for A. Use Table 2 above to check whether your equation works.

**Answer:** Since amplitude is always positive, we need to use absolute value: \( \text{Amplitude} = |A| \)

**Challenger questions:**

Notice that slider A allows only whole numbers. Let us see if our equation for amplitude from Question 12 works when A is a fraction.

First, let’s see what happens to the functions when A is a fraction between 0 and 1.

- Click on the checkbox to toggle **Allow Fraction Values for A**

13. Move slider A from A= 1 to A= ½. What type of transformation does this represent?

**Answer:** Moving A from A= 1 to A= ½ stretches the function vertically (a vertical dilation).

14. Set A= ½. Write the equation for both the sine and cosine function below.

**Answer:** \( y = \frac{1}{2} \sin(x) \) and \( y = \frac{1}{2} \cos(x) \)

15. What is the amplitude of both sine and cosine under this vertical dilation?

**Answer:** The amplitude of these functions is \( \text{amplitude} = \frac{1}{4} \)

16. Set A= ¼. Write the equation for both the sine and cosine function below and then find the amplitude of these functions. Fill in **row2** of the table below.

**Answer:** \( y = \frac{1}{4} \sin(x) \) and \( y = \frac{1}{4} \cos(x) \) and the amplitude of these functions is \( \text{ampl} = \frac{1}{4} \).

17. Now using your equation for \( P \) from Question 12, calculate the amplitude \( P \) for A= ½ and A= ¼. Fill in **row3** of the table below and check that row2 and row3 are the same!

### Table 2: Positive and Negative Whole Numbers for A.

<table>
<thead>
<tr>
<th>Dilation Factor</th>
<th>A = 1</th>
<th>A = 2</th>
<th>A = 4</th>
<th>A = -1</th>
<th>A = -2</th>
<th>A = -4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>( \text{ampl} = 1 )</td>
<td>( \text{ampl} = 2 )</td>
<td>( \text{ampl} = 4 )</td>
<td>( \text{ampl} = 1 )</td>
<td>( \text{ampl} = 2 )</td>
<td>( \text{ampl} = 4 )</td>
</tr>
<tr>
<td>Amplitude using Equation</td>
<td>( \text{ampl} =</td>
<td>A</td>
<td>) = (</td>
<td>1</td>
<td>= 1 )</td>
<td>( \text{ampl} =</td>
</tr>
</tbody>
</table>

### Table 3: Positive Fractions for A.

<table>
<thead>
<tr>
<th>Dilation Factor</th>
<th>A = 1</th>
<th>A = ( \frac{1}{2} )</th>
<th>A = ( \frac{1}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>( \text{amplitude} = 1 )</td>
<td>( \text{amplitude} = \frac{1}{2} )</td>
<td>( \text{amplitude} = \frac{1}{4} )</td>
</tr>
<tr>
<td>Amplitude using Equation</td>
<td>( \text{amplitude} =</td>
<td>A</td>
<td>) = (</td>
</tr>
</tbody>
</table>
18. Let’s look at \( A \) between -1 and 0. Complete Table 4 with \( A = -1, A = -\frac{1}{2} \) and \( A = -\frac{1}{4} \). Check that row 2 and row 3 are the same.

### Table 4: Negative Fractions for \( A \)

<table>
<thead>
<tr>
<th>Dilation Factor</th>
<th>( A = -1 )</th>
<th>( A = -\frac{1}{2} )</th>
<th>( A = -\frac{1}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>amplitude = 1</td>
<td>amplitude = ( \frac{1}{2} )</td>
<td>amplitude = ( \frac{1}{4} )</td>
</tr>
<tr>
<td>Amplitude using Equation</td>
<td>amplitude = (</td>
<td>A</td>
<td>=</td>
</tr>
</tbody>
</table>

### Conclusions (cross out incorrect responses in blocks and fill in blanks):

1. Changing the value of \( A \) dilates (stretches or compresses) the functions \( y = \text{Asin}(x) \) and \( y = \text{Acos}(x) \) along the **horizontal** axis.

2. When \( |A| > 1 \), the functions \( y = \text{Asin}(x) \) and \( y = \text{Acos}(x) \) are **stretched** and the amplitude \( A \) of these functions is **amplitude > 1**.

3. When the **amplitude < 1** of the functions \( y = \text{Asin}(x) \) and \( y = \text{Acos}(x) \), this means that \( 0 < |A| < 1 \) and the functions are **stretched** vertically.

4. The amplitude of \( y = \text{Asin}(x) \) and \( y = \text{Acos}(x) \) is determined by the formula

\[
\text{Amplitude} = |A|
\]