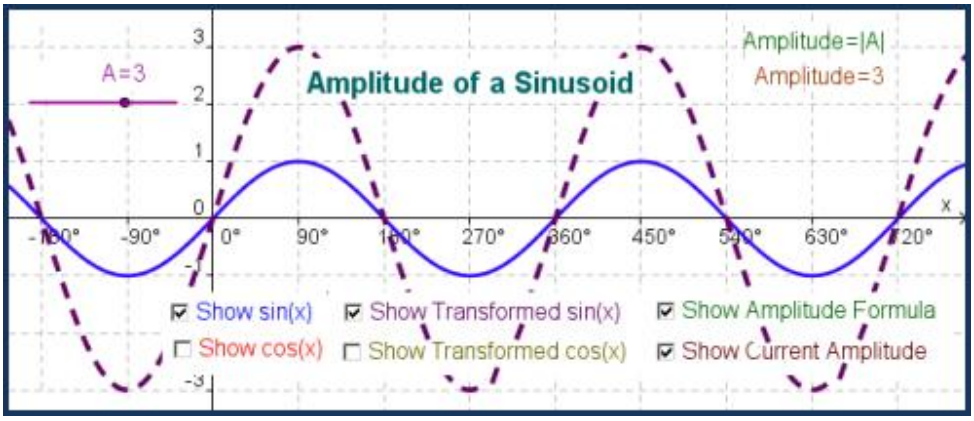
Amplitude and Vertical Dilation of Sine and Cosine Functions

In this activity, you will discover the relationship between the vertical dilation A and the amplitude of: $y = A\sin(x)$ and $y = A\cos(x)$

Applet: Amplitude_Vertical_Dilation

Terminology: Vertical dilation *stretches* or *compresses* the function along the y-axis.



Before you begin, check that: A=1 and that the checkbox "Allow Fractions for A" is *unchecked*.

Questions:

1. Write down the parent functions of $y = A\sin(x)$ and $y = A\cos(x)$. What is the value of A and what is the amplitude of these parent functions?

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2. Move slider A in any direction (A≠0). What type of transformation does this represent?

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3. What happens to y = Asin(x) and to y = Acos(x) when A increases in the positive direction?

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4. Set A = 2. Write the equations for both the sine and cosine functions below.

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5. What is the amplitude of both sine and cosine under this vertical dilation?

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6. Set A = 4. Write the equations for both the sine and cosine functions below.

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7. What is the amplitude of both sine and cosine under this vertical dilation?

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There seems to be a relationship between the amplitude of a sinusoidal function and A.
Let's see if you can see the pattern.

Table 1: Positive Whole Numbers for A.

Dilation Factor	A = 1	A = 2	A = 4
Amplitude	*amplitude* = 1	*amplitude* = 2	amplitude = 4

8. Write an equation for the *amplitude* in terms of A when A>0.

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9. Review question: Set A=1 and then A=-1. What changes in the graph of the sine function? What changes in the cosine function? Complete the equations: -sin(x)=? and -cos(x)=?

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10. Let's see what happens when A is negative? Set A = –1. What is the amplitude of the sine and cosine function? What is the amplitude of the sine and cosine function when A = –2 and when A = –4? Fill in row2 of Table 2 below.

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11. Does the *amplitude* change between when A is positive and A is negative?

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Table 2: Positive and Negative Whole Numbers for A.

Dilation Factor	A = 1	A = 2	A = 4	A = -1	A = -2	A = -4
Amplitude	*ampl* = 1	*ampl*= 2	*ampl*= 4	*ampl*=	*ampl*=	*ampl*=
Amplitude using Equation						

12. Modify your equation for amplitude from Question 8 to include both positive and negative values for A. You will have to change your expression for A. Use Table 2 above to check whether your equation works.

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Challenger questions:

Notice that slider A allows only whole numbers. Let us see if our equation for *amplitude* from Question 12 works when A is a fraction.

First, let's see what happens to the functions when A is a fraction between 0 and 1.

- Click on the checkbox to toggle *Allow Fraction Values for A*

13. Move slider A from A= 1 to A= ½. What type of transformation does this represent?

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14. Set A= ½. Write the equation for both the sine and cosine function below.

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15. What is the amplitude of both sine and cosine under this vertical dilation?

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16. Set A= ¼. Write the equation for both the sine and cosine function below and then find the amplitude of these functions. Fill in row2 of the table below.

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17. Now using your equation for A from Question 12, calculate the amplitude A for A= ½ and A= ¼. Fill in row3 of the table below and check that row2 and row3 are the same!

Table 3: Positive Fractions for A

Dilation Factor	A = 1	A = ½	A = ¼
Amplitude	*amplitude*= 1	*amplitude* =	*amplitude* =
Amplitude using Equation			

18. Let's look at A between -1 and 0. Complete Table 4 with A=-1, A=-½ and A=-¼. Check that row2 and row3 are the same.

Table 4: Negative Fractions for A

Dilation Factor	A = -1	A = -½	A = -¼
Amplitude	*amplitude* = 1	*amplitude* =	*amplitude* =
Amplitude using Equation			

Conclusions (cross out incorrect responses in blocks and fill in blanks):

1. Changing the value of A dilates (stretches or compresses) the functions $y = A\sin(x)$ and $y = A\cos(x)$ along the horizontal vertical axis.
2. When $|A|>1$, the functions $y = A\sin(x)$ and $y = A\cos(x)$ are stretched compressed and the amplitude A of these functions is amplitude > 1 amplitude < 1.
3. When the amplitude < 1 of the functions $y = A\sin(x)$ and $y = A\cos(x)$, this means that $|A|>1$ $0<|A|<1$ and the functions are stretched compressed vertically.
4. The amplitude of $y = A\sin(x)$ and $y = A\cos(x)$ is determined by the formula

amplitude =