

Amplitude and Vertical Dilation of Sine and Cosine Functions

In this activity, you will discover the relationship between the vertical dilation A and the amplitude of: $y = A\sin(x)$ and $y = A\cos(x)$

Applet: [Amplitude_Vertical_Dilation](#)

Terminology: **Vertical dilation** *stretches* or *compresses* the function along the y-axis.

Before you begin, check that: $A=1$ and that the checkbox "Allow Fractions for A " is *unchecked*.

- Write down the parent functions of $y = A\sin(x)$ and $y = A\cos(x)$. What is the value of A and what is the amplitude of these parent functions?
- Move slider A in any direction ($A \neq 0$). What type of transformation does this represent?
- What happens to $y = A\sin(x)$ and to $y = A\cos(x)$ when A increases in the positive direction?
- Set $A = 2$. Write the equations for both the sine and cosine functions below.
- What is the amplitude of both sine and cosine under this vertical dilation?
- Set $A = 4$. Write the equations for both the sine and cosine functions below.
- What is the amplitude of both sine and cosine under this vertical dilation?

There seems to be a relationship between the amplitude of a sinusoidal function and A .

Let's see if you can see the pattern.

Dilation Factor	$A = 1$	$A = 2$	$A = 4$
Amplitude	<i>amplitude</i> = 1	<i>amplitude</i> = 2	<i>amplitude</i> = 4

- Write an equation for the *amplitude* in terms of A when $A > 0$.
- Review question: Set $A=1$ and then $A=-1$. What changes in the graph of the sine function? What changes in the cosine function? Complete the equations: $-\sin(x)=?$ and $-\cos(x)=?$
- Let's see what happens when A is negative? Set $A = -1$. What is the amplitude of the sine and cosine function? What is the amplitude of the sine and cosine function when $A = -2$ and when $A = -4$? Fill in row 2 of the table below.
- Does the *amplitude* change between when A is positive and A is negative?
- Modify your equation from Question 8 to include both positive and negative values for A . You will have to change your expression for A . Use the table below to check whether your equation works.

Dilation Factor	$A = 1$	$A = 2$	$A = 4$	$A = -1$	$A = -2$	$A = -4$
Amplitude	<i>ampl</i> = 1	<i>ampl</i> = 2	<i>ampl</i> = 4	<i>ampl</i> =	<i>ampl</i> =	<i>ampl</i> =
Amplitude using Equation						

Challenger questions:

Notice that slider A allows only whole numbers. Let us see if our equation for *amplitude* from Question 12 works when A is a fraction.

First, let's see what happens to the functions when A is a fraction between 0 and 1.

- Click on the checkbox to toggle *Allow Fraction Values for A*

13. Move slider A from $A = 1$ to $A = \frac{1}{2}$. What type of transformation does this represent?
14. Set $A = \frac{1}{2}$. Write the equation for both the sine and cosine function below.
15. What is the amplitude of both sine and cosine under this vertical dilation?
16. Set $A = \frac{1}{4}$. Write the equation for both the sine and cosine function below and then find the amplitude of these functions. Fill in row2 of the table below.
17. Now using your equation for A from Question 12, calculate the amplitude A for $A = \frac{1}{2}$ and $A = \frac{1}{4}$. Fill in row3 of the table below and check that row2 and row3 are the same!

Dilation Factor	$A = 1$	$A = \frac{1}{2}$	$A = \frac{1}{4}$
Amplitude	<i>amplitude</i> = 1	<i>amplitude</i> =	<i>amplitude</i> =
Amplitude using Equation			

18. Let's look at A between -1 and 0. Complete Table 4 with $A = -1$, $A = -\frac{1}{2}$ and $A = -\frac{1}{4}$. Check that row2 and row3 are the same.

Dilation Factor	$A = -1$	$A = -\frac{1}{2}$	$A = -\frac{1}{4}$
Amplitude	<i>amplitude</i> = 1	<i>amplitude</i> =	<i>amplitude</i> =
Amplitude using Equation			

Conclusions (cross out incorrect responses in blocks and fill in blanks):

1. Changing the value of A dilates (stretches or compresses) the functions $y = A\sin(x)$ and $y = A\cos(x)$ along the horizontal vertical axis.
2. When $|A| > 1$, the functions $y = A\sin(x)$ and $y = A\cos(x)$ are stretched compressed and the amplitude A of these functions is amplitude > 1 amplitude < 1.
3. When the amplitude < 1 of the functions $y = A\sin(x)$ and $y = A\cos(x)$, this means that $|A| > 1$ $0 < |A| < 1$ and the functions are stretched compressed vertically.
4. The amplitude of $y = A\sin(x)$ and $y = A\cos(x)$ is determined by the formula

$amplitude =$