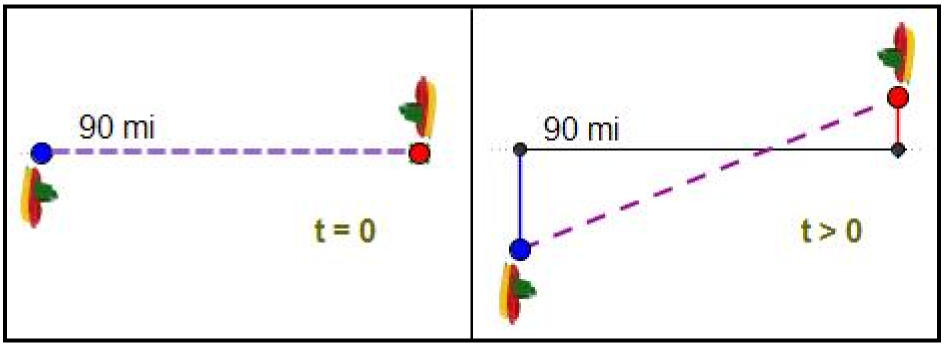
Related Rates Ship Simulator[1]

Statement of the Problem: At noon, ship A is 90 km west of ship B. Ship A is sailing south at 40 km/h and ship B is sailing north at 20 km/h. How fast is the distance between the ships changing at 2:00 PM?

We make a sketch of what is happening so that we can place the problem on a graph.



What we want: The purple line is the distance between the boats. We want to know how fast the length of this line is increasing at time t=2hrs.

Setup: From the sketch, we decide to make the left point at A=(0,0) and the right point B=(90,0).

Brief Directions for the making the simulator.

1. Select and click on (0,0) so create point A=(0,0) and then click anywhere on the **positive x-axis** to create point B=Point[xAxis].

   In this way, B is a *movable point on the positive x-axis*. We want B to be movable for 2 reasons – for testing purposes and to allow the simulator more flexibility. We want B tied to the x-axis to keep everything East-West.

2. Click anywhere in the drawing pad and use the mouse scroll button to Zoom out and then click and drag B to (90,0).

   For the moment, we will not think about scaling x-axis and y-axis.

3. Click in the Input Bar (bottom left) and input t=2, va=40 and vb=20. All are numbers. In the Algebra window, right-click on each of these and select "Show object". This makes them slider variables. Later we can adjust the intervals and increments.

   Making these numbers into sliders makes the simulator both testable and versatile.

4. On our sketch, the position of boat A is the blue point A_t= A+(0,-va*t) and the positon of boat B is the red point **B_t =B+(0,vb*t)**. Create these points in the Input Bar.

5. We draw "travel lines", that is, using we create a line segment a from A to A_t and line segment b from B to B_t . From here it is easy to see that the ***distance*** between the boats is the line segment c from A_t to B_t. Finally, we draw the line segment d joining A and B. We get the picture shown in sketch.

   Notice that we have not yet defined any functions so with what we have, we cannot find the rate at which the length of line segment c is increasing.

[1] This simulator first appeared in the article by Fahlberg-Stojanovska, L and Stojanovski, V. *GeoGebra: Freedom to Explore and Learn*. Teaching Mathematics and its Application, Oxford Journals, June 2009.

6. It is however easy to see that the line segment c is the hypotenuse of a triangle with side **d** and side **a**+**b** so using Pythagoras' theorem, we define the "distance function" $s(x)$.

Function discussion: As always, ***x*** is the variable for **time**. We have already defined t as a *number* and t always has a "current" value that is given in the Algebra window at left.

In fact, the $s(t)$ which is value of $s(x)$ at $x=t$ should give the "current" distance between the boats, that is, the length of the segment c. This is a good way to check our function.

Notice that d does *not* depend on time – it is the initial distance beteen the boats so ***fixed number*** for any particular problem. (For the posed problem d=90.) So we can use d directly in the formula. However, **a** and **b** depend on time so we use their formulas: length of a = va*x and length of b = vb*x.

This gives us $s(x) = \sqrt{d^2 + (va \cdot x + vb \cdot x)^2} = \sqrt{d^2 + (va + vb)^2 x^2}$

7. Input s(x)=sqrt(d^2+(va+vb)^2*x^2)
8. Input dis=s(t) and check that dis=c.

In the Algebra window, GeoGebra will show $s(x)$ with the current values of d, va and vb. Changing these values (via the sliders or moving point B) will change this formula.

With this, we have built the simulator.

Finding the answer: We want the *change in the distance s(x) with respect to time x*. That is, we want $s'(x)$, so we

9. Input sDer=Derivative[s].

GeoGebra will calculate $s'(x)$ and write the resulting function sDer(x) with the current values of d, va and vb in the Algebra window.

The answer to the problem is the value of the derivative when $t=2$. So we input:

10. ans=sDer(t) and then slide t to 2.

This answer – that is, the value of ans - can be read in the Algebraic window at the left. For the given problem, ans=48 [km/h].

Thinking and calculus: This little simulator is fully functional; one can slide the values for time and speeds as well as move point B to change the east-west distance. We can check the validity of our simulator by posing some good questions. Finally, do the mathematics. That is, write down the function, find the derivative, calculate its value for the given t and check it against the simulator.

Sample good questions: *Make the simulator simulate both boats starting at the same position. Set the speeds and animate the slider t from 0 to 5. Describe the rate of change in the distance between the boats. Change the speeds and test again. What can you conclude?* First, we must think about moving B onto A. Then, by moving t, we see that the 2 ships are just moving directly away from each other. We should notice that the "answer" is constant. Check the function and see **d=0** so $s(x)=(va+vb)x$ (no square root). The *derivative is constant* sDer(x)=va+vb, so the answer is just the sum of the 2 speeds. This makes sense. We have checked that our simulator works properly in a "known" situation.

Always think about the whole process: Focus on what is really happening in the problem – crucial to visualizing and understanding the mathematics. Secondly, use your dynamic simulator to check that "known" cases are being solved correctly. If it doesn't, there must be a problem with our simulator or our thinking. This is a vital – but often severely neglected – step in any logical thinking process. But it is easy to check with a simulator – so do it!