Simulator Kit: CarRace (Student's kit)

In your kit:

1. GeoGebra files: Ready-to-use projector sized simulators: car1.ggb, car1P.ggb and car2P.ggb
2. Images 6: redcar20.gif, bluecar20.gif, redcar15.gif, bluecar15.gif, redcar10.gif, bluecar10.gif
3. CarRaceSimulator.pdf (this file) and CarRaceSimulatorEd.pdf (educator's pages)

Scenario: Everyday Rila drives her red car the 100 mile distance across the flat plains of Flatlandia between the cities AbsolutelyNowhere and BeyondBoring. She drives the whole route at a constant speed using cruise control. A stopwatch displays the number of hours past and a GPS displays the number of miles the car is from AbsolutelyNowhere.

Goals: (a) To create an animated simulation of a car driving at a constant speed the 100 mile distance from point A to point B with a spreadsheet showing the time past since resetting the stopwatch and the distance the car is from point A.
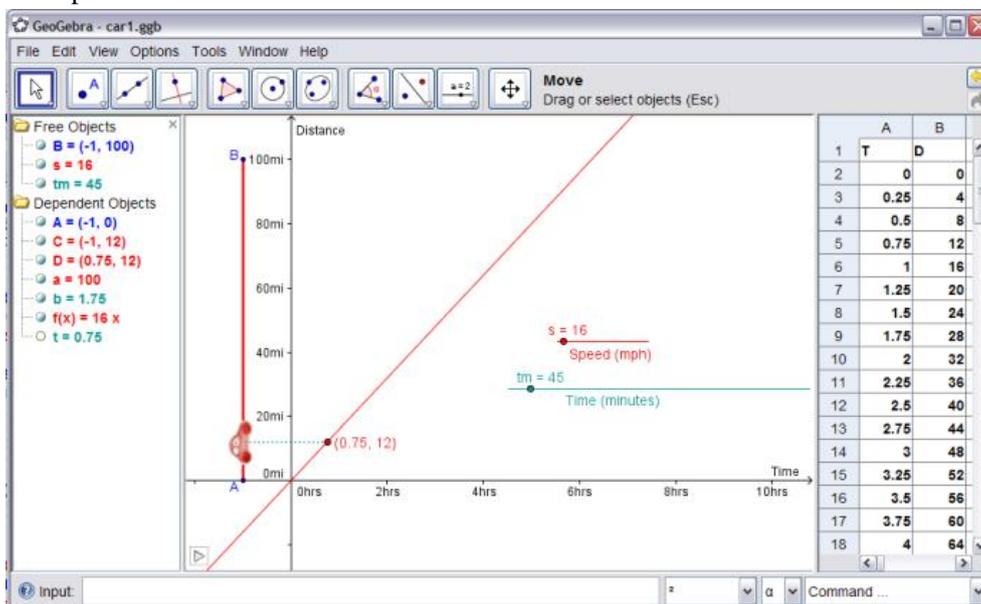
(b) To expand this simulation to allow the car to have a variable starting position.
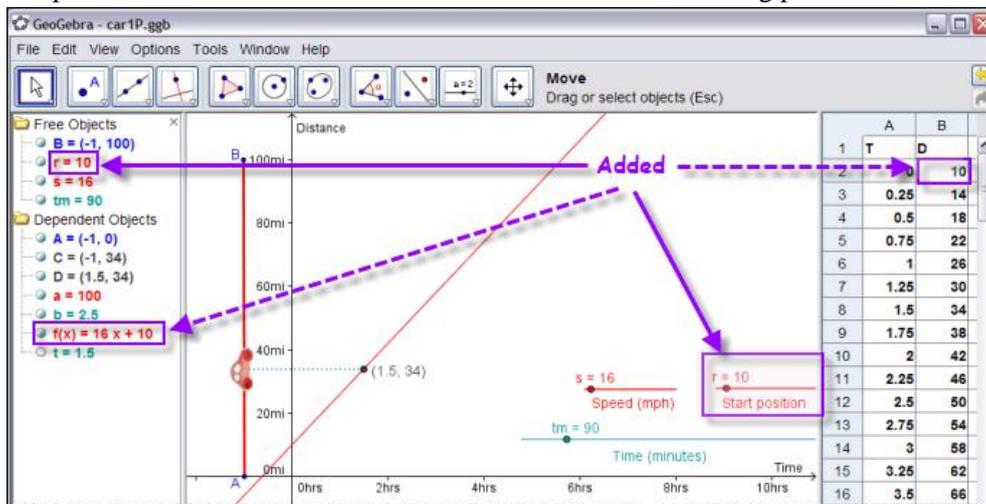
(c) To expand this simulation to include 2 cars with variable starting positions.
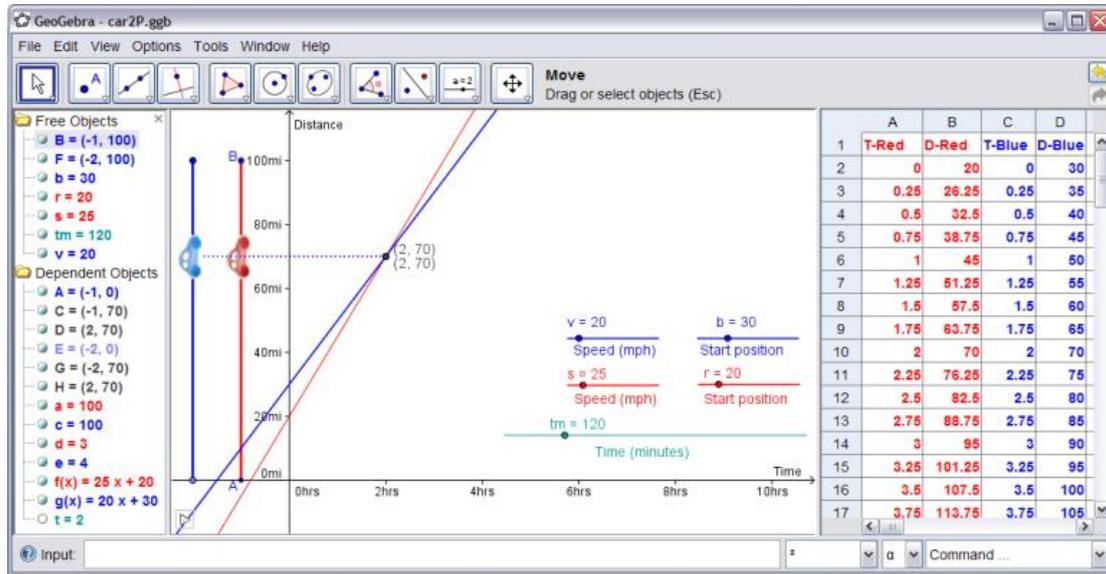
1. Directions for ONE car simulator (red car from A to B = car1.ggb)

Typical Question for this Simulator: Today Rila leaves AbsolutelyNowhere at 9AM. She resets the stopwatch to 0 and she sets her cruise control at 16 mph. What 2 numbers are displayed at 9:45AM?
Starting when she resets the stopwatch to 0, every 15 minutes, the car prints out the numbers on the display. What does the printout look like at 9:45AM? What time is it when Rila arrives in BeyondBoring?

1. Open GeoGebra (double-click on GeoGebra icon ⭕).

   Make sure the Algebra View is visible at left (View→Algebra View), the spreadsheet is visible is at right (View→Spreadsheet) and the Input bar is visible at bottom (View→Input bar).

   This is a horizontal splitting (View→Horizontal Splitting)

   Our defaults are: Options->Rounding->**2 decimal places**, Options->Labeling->**No new objects**.

2. Scale and label the axis.
   The horizontal axis (x-axis) will be the variable Time (in hours)
   The vertical axis (y-axis) will be the variable Distance (in miles)

We plan on a **minimum speed of 10** mph and a **maximum distance of 100** miles. From this, we need an **x-axis from 0-10** hours and **y-axis from 0-100** miles

 A square scale would be 1:10, but **we will use 1:20** because most monitors are wide (not square).

Before you start make sure that the Grid is turned on (View→Grid). Later you will turn it off, but for now it will help you build the simulation. Right-click in the drawing pad. A drop-down menu will open.

a. Go down to x-axis:y-axis and click.
   Another drop-down menu will open.

b. Go up to 1:20 and click .

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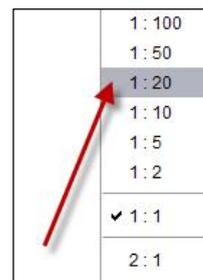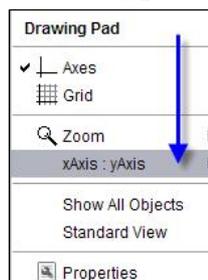
c. Click on *Move Drawing Pad* tool and use your mouse scroll button to zoom in and out until the drawing pad shows the x-axis from -2 to 11 and y-axis from -40 to 120.

d. Right-click again in the drawing pad and choose Properties. A dialog box will open.

  i.   On the xAxis tab, type in Unit: **hrs** and Label: **Time**

  ii. Click on the yAxis tab and type in Unit: **mi** and Label: **Distance**
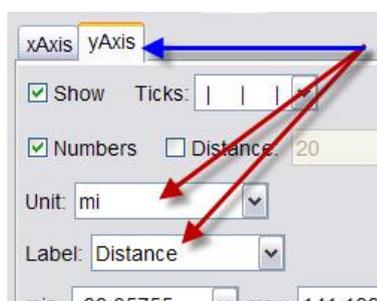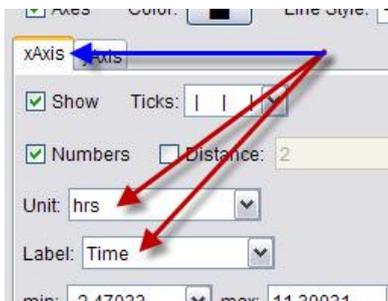
3. Make a line segment AB

  a. Select segment tool
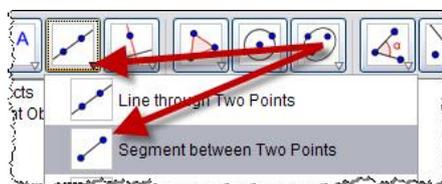
  b. In the drawing pad, click once on (-1,0) and then click again on (-1,100)
      (This vertical line segment will show the small car move along for 100 miles.)

      Look in the Algebra View (window at left)

      Notice objects: point **B(-1,100)** and point **A(-1,0)** and segment **a=100**.

      Notice that point A(-1,0) and the segment a are in the "dependent" category. This means they are dependent on something: point **A is light-blue (semi-dependent)**[1] because it is "tied to" the x-axis and **segment a is black** because it is dependent on the points A and B).

4. Make slider for time t. This will be our animator.

    We want t to be in minutes. If we have 10 hours, how many minutes?

  a. Select slider tool

  b. In the drawing pad, click once towards the bottom right corner, but above the horizontal axis (once created you can move the slider using the *Move* tool). A dialog box will open.

  i.   First, type tm (a small t and a small m) to change the **name of the slider to** tm.

  ii. Click in min box and change to 0.

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[1] If you want A to be free, right-click on it and choose Properties. On the Basics tab, type (-1,0) in place of Point[xAxis].

  iii. Click in max box and change to 600. Why 600?

  iv. Click in increment box and change to 1 (for easy manual manipulation)

  v. Click on the Slider tab and change slider width to 300.

  vi.
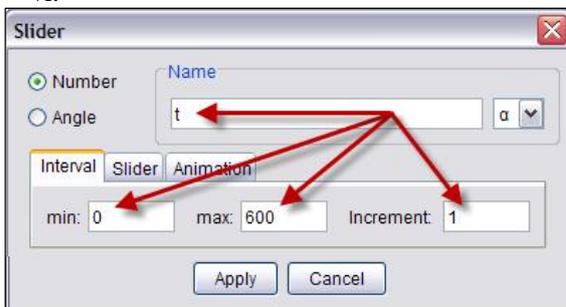
                   vii. Click on Apply.

  viii. Use the ⬚ to click and drag the slider into bottom right corner.

  ix. Then use ⬚ to drag the slider to t=**30** (half an hour) or simply click in the Input Bar (bottom left) and type: tm=30 and **hit Enter**.
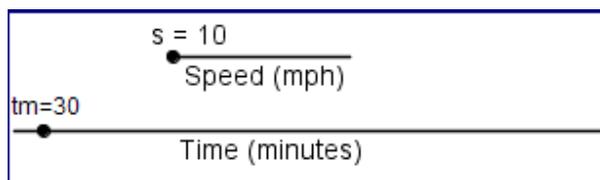
5. Make the **s** slider for the speed of the car.

 a. Make sure the slider tool is selected, click just above the tm slider. In the dialog box, make the following changes:

  • First, type s (a small s) to change the **name of the slider to** s.

  • Click in min box and change to 10.

  • Click in max box and change to 100.

  • Click in increment box and change to 1 (for easy manual manipulation)

  • Click on the Slider tab and change slider width to 90.

   Sliders work better if their width is a multiple of the interval. Here 100-10=90.

6. Make texts for the sliders.

 a. Select the text tool ABC

 b. Click under the slider for tm and type: "Time (minutes)". Click on OK.

 c. Click under the slider for s and type: "Speed (mph)". Click on OK.

  (you can always adjust the location of the text by using the Move tool)

7. Make the function f(x) for D=ST.

 a. Click in the Input bar (bottom left) and type: f(x)=s*x and hit Enter**.** (Remember –case-sensitive.)

  CRUCIAL POINT: f(x) is the *whole* function, f(t) will be the value of this function at time t.

    f(x) **is the whole road,** f(t) is where the car is on the road **at time** t

8. Make the moving point for the car C and the moving point on the function D.

   a. Remember the speed s is in mph and the unit for Time on the horizontal axis is in hours. However the unit for tm is in minutes so we need to divide tm by 60 to adjust.

   We will make a new variable *t* for time in hours.
   Click in the Input bar (bottom left) and type: t=tm/60 and hit Enter.
      (or mark the text, hit Ctrl+C, click in the input bar, hit Ctrl+V and then hit Enter).

   b. Now we do not want the car to drive past the city BeyondBoring which has a y-coordinate of 100. So this means that the yPosition of the red car should not be more than 100.
   Click in the Input bar and type: yPosR=Min[f(t),100] and hit Enter.

   c. Click in the Input bar and type: C=(-1, yPosR) and hit Enter.

   d. Click in the Input bar and type (or copy&paste): D=(t,yPosR)   and hit Enter.
   This is a crucial point of understanding: D is the point where the *horizontal coordinate is the time* t and the *vertical coordinate is distance*, that is: f(t)=s*t (speed multiplied by time). It is worthwhile to spend time on understanding this point.

9. Connect and animate points C and D.

   a. Select the segment tool and click on C and then on D.

   b. Select the *Move* tool ⬚ and then click and drag on the slider point tm to manually animate the points. Not to be boring, but notice that *at all times* the vertical coordinates of the points C and D are the same - both are f(t). Again, this is the distance of the car from A!

   c. Right-click on tm (either the slider point or the value) and from the drop-down menu, select Animation On. The points C and D on the segment should start to move from end to end.

   The pause ⬚ and play ▷ buttons are always at the bottom left of drawing pad.

10. Add the little car.

   a. Select the Insert Image tool ⬚. Click on point C. In the dialog box select redcar20.gif. Notice that as an image, the car is stretched vertically (by a factor of 20).
   Why does it look normal in the geogebra file? Remember our scaling!
   Unfortunately C is the bottom left corner of the image.

   b. To center the car over C, right-click on the image and select properties from the drop-down menu. Click on the Position tab and for Corner 1 copy and paste in: C-(0.2,5).

   c. If you don't want to see the point, click on the Advanced tab and change the Layer to 1.

   d. Click on Close.

11. Create the spreadsheet table of values T and D for 15 minute intervals.

   a. Click in cell A1 and type: T and hit Tab. The cursor will move to cell B1.

   b. Type: D and hit Tab or Enter.

    c. Click in cell A2 and type: 0 (a zero) and hit Tab. The cursor will move to cell B2.

    d. Type: =f(A2) and hit Enter. (Remember –case-sensitive!) This number will be 0 since the distance the car has traveled at time t=0 is zero!

   e. Click in cell A3 and type: =A2+0.25 (to add 15 minutes to the time above). Hit Enter.
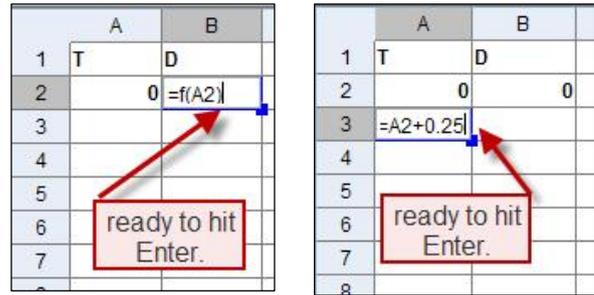
Note: if you want to be able to change the time intervals in the spreadsheet, first define a number tint. That is, click in the input bar and type: tint=0.25 and hit Enter. Then in cell A3, type: =A2+tint and hit Tab. To change the value of tint to (say) half-hour intervals, simply click in the input bar and type: tint=0.5 and hit Enter. All of the values in the spreadsheet will be updates.

    f. Copying cells and relative addresses.

We want to copy the formulas we have typed in B2 and A3 to the cells below , but with the proper change of address. That is, in cell B2 we have =f(A2) so in B3 we want =f(A3) . As in Excel, GeoGebra will do this automatically.

- Click in cell B2. Click and drag little blue square down one cell. (see below) This copies the distance formula f(A2) but changes the address relatively to f(A3) to get distance for the new time.
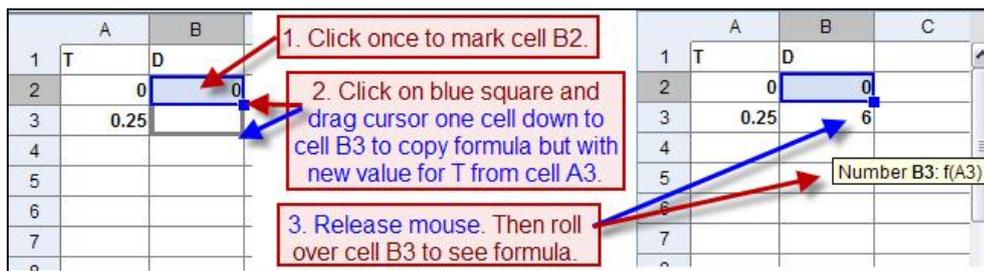
- Now we are ready to copy both formulas (with addresses of cell changing relatively). Click in cell A3 and drag cursor right one cell (Both cells A3 and B3 should be marked). Click and drag little blue square down to cell B42 (see right).
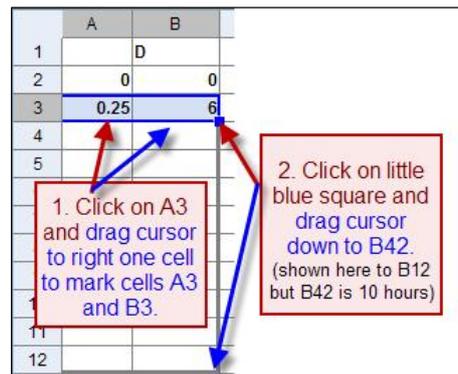
12. Save your file!

We recommend you save this file as: car1.ggb

Some mathematics to think about …

First do the problem with the simulator. Then write down the answers with mathematics.

- Set s = 25mph, r = 0 miles. How long does it take the car to reach the endpoint (at 100 miles)? Find your answer both in minutes and in hours on the simulation. Can you find (T,D) on the function (the red line) in the drawing pad and in the spreadsheet for D=100 miles?

2. Simulator with Variable Starting Position (red car from *start r* to B = car1P.ggb)

1. Make the **r** slider for the starting position of the car.

   a. Select the **slider** tool [⊞]
   b. In the drawing pad, click towards the bottom right corner.
      i. First, type r (a small r) to change the **name of the slider to** r.
      ii. Click in min box and change to 0.
      iii. Click in max box and change to 100.
      iv. Click in increment box and change to 1 (for easy manual manipulation)
      v. Click on Apply.
   c. Use the [↖] to click and drag the slider to **20**.

2. Make text for the slider.

   a. Select the Insert **text** tool [ABC]
   b. Click under the slider for t and type: "Starting Position (miles)". Click on OK.

3. Add the starting position to the function f(x)=s*x+r.

   a. Right-click on f(x) (either in the algebra window or on the line itself).
   b. Select Properties from drop-down menu.
   c. On the Basics tab, in the Definition field click at the end and add "+r".

Basic	~~olor~~ ~~Style~~ ~~Advanc~~
Name:	f
Definition:	f(x) = s x + r
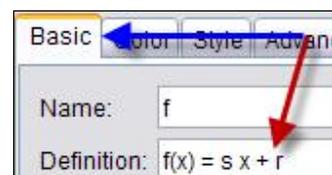
   Notice that the spreadsheet automatically is okay since the cell entries are defined using the function and we have change it to include the starting position.

4. Save your file!

   We recommend you save this file as: car1P.ggb

Change the speed and starting position and manually animate your car by sliding tm.
See if you can find out how long it will take to reach the end point.

A little more mathematics to think about …

First do each problem with the simulator. Then write down the answers with mathematics.

- Set s = 20mph, r = 20 miles. How long does it take the car to reach the endpoint?
- Set r = 32 miles. How fast must red car go to reach the endpoint in 4 hours?
- Set s = 45mph. Red car reached the endpoint in 1hr and 52 minutes. Where did he start?

3. Simulator with TWO cars (red and blue cars from *start positions* to B = car2P.ggb)

Let's say we want to also have a blue car travelling to point B.

1. We suggest you first make everything from the first car red.

   a. Right-click on **r** and choose Properties from the drop-down menu.

   b. Click on the color tab and choose a red tile.

   c. In the left panel of the Properties dialog box, click on each of function **f**, number **s**, points **C** and **D**, segment **a** and **text2** and **text 3** and then click on the same tile (now in recent).

   d. While you are there, you can make numbers **t, tm** and **text1** a color like dark green.

   e. When you are done, click on Close.

   f. Select all the cells in the spreadsheet that we have used (A1-B42). Right-click, choose Properties. Click on the color tab and choose a red tile.

2. Now, make sliders **v** for the speed and **b** for the position of the blue car. Label them with text.

3. Make the function **g(x)** for the blue car like **f(x)** above.

4. Make a line segment from (-2,0) to (-2,100) for driving the blue car.

5. Create **yPosB** and 2 points **G and H** for the blue car like **C and D** above.

6. Draw a line segment between **G and H**.

7. Insert an image of the blue car over point **H**.

8. Test your construction by clicking and dragging t – both cars should move.

9. Create spreadsheet columns C and D like A and B using the function g in place of f.

10. Select all the cells in the spreadsheet for the blue car and make them blue.

11. Make v, b, g(x), G, H and text4 and text5 blue.

12. Save your file! We recommend you save this file as: car2P.ggb

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Change the speeds and starting positions and race your cars.

See if you can find out how long each takes to reach the end point.

See if you can find out if (and when) they pass each other.

A little more mathematics to think about …

First do each problem with the simulator. Then write down the answers with mathematics.

- Set s = 25mph, r = 20 miles, v=25 mph and b =10 miles. Which car starts out ahead? Do they meet before getting to the endpoint (at 100 miles). What can you say about the distance between the two cars?

- Set s = 25mph, r = 20 miles, v=20 mph and b =30 miles. Which car starts out ahead? Do they meet before getting to the endpoint (at 100 miles). If so, how long and far have they travelled when the meet?

- Set r =40 miles and b =24 miles. What speeds do you need to set for the cars to meet at the endpoint (at 100 miles) at t =2hrs.

- Make up a question and see if your friend can solve it.