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In this activity, we will explore transformations of figures using matrices.

Open the applet: click here.

Take a few moments and explore the applet. Use the questions below to help guide your exploration.

1. What does matrix *M* represent?

   Matrix *M* represents the coordinates of the 4 vertices of the pre-image, i.e. the original quadrilateral.

2. What does matrix *M′* represent?

   Matrix *M'* represents the coordinates of the 4 vertices of the image of the original quadrilateral after being transformed (translated) by the matrix T.

3. What does matrix *T* represent?

   Matrix T represents the transformation matrix, i.e. the coordinates by which each vertex of M is to be translated.

Move slider *a* in the positive direction to different values. Each time look at how the three matrices *M*, *M′* and *T* change.

4. Move slider *a* such that *a*=5. Record each matrix *M, M′* and *T* below.

$$a=5,\ M=\begin{bmatrix} 0 & -1 & -4 & -4 \\ 0 & 4 & 2 & 0 \end{bmatrix}\quad T=\begin{bmatrix} 5 & 5 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}\ \text{and}\ M'=\begin{bmatrix} 5 & 4 & 1 & 1 \\ 0 & 4 & 2 & 0 \end{bmatrix}$$

5. What do you notice about the value of slider *a* and its relationship to the translation matrix T?

   The value of slider *a=5* is the values of the elements of the first row of T (that is, the x-coordinates).

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Move the slider from *a*=5 to *a*=4.

6. Write the matrices *M*, *M'* and *T* when *a* = 4 below. What values in *M'* changed from those in problem 5? By how many units?

$$a=4,\ M=\begin{bmatrix}0 & -1 & -4 & -4\\ 0 & 4 & 2 & 0\end{bmatrix}\quad T=\begin{bmatrix}4 & 4 & 4 & 4\\ 0 & 0 & 0 & 0\end{bmatrix}\ \text{and}\ M'=\begin{bmatrix}4 & 3 & 0 & 0\\ 0 & 4 & 2 & 0\end{bmatrix}$$

The top row of M' has changed. The values have decrease by 1 just as *a* decreased by 1 from 5 to 4.

7. Write a matrix equation that shows how *M*, *T* and *M'* are related when *a*=4.

$$\begin{bmatrix}0 & -1 & -4 & -4\\ 0 & 4 & 2 & 0\end{bmatrix}+\begin{bmatrix}4 & 4 & 4 & 4\\ 0 & 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}4 & 3 & 0 & 0\\ 0 & 4 & 2 & 0\end{bmatrix}$$

8. What if *a* were negative? Which direction does the pre-image translate?

If *a* is negative then the pre-image translates *to the left*.

9. Write a matrix equation showing the relationship between *M'* and *T* that represents the translation when *a* = -3.

$$\begin{bmatrix}0 & -1 & -4 & -4\\ 0 & 4 & 2 & 0\end{bmatrix}+\begin{bmatrix}-3 & -3 & -3 & -3\\ 0 & 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}-3 & -4 & -7 & -7\\ 0 & 4 & 2 & 0\end{bmatrix}$$

10. Which coordinate does *a* seem to affect? Make a conjecture on what coordinate *b* will affect.

The value of a affects the x-coordinates of M'.

Conjecture: the value of *b* will affect the y-coordinates of M'.

Set *a* = 0 and *b* = 5.

11. Write the matrix equation that shows the relationship between *M*, *M'* and *T* below.

$$\begin{bmatrix}0 & -1 & -4 & -4\\ 0 & 4 & 2 & 0\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0\\ 5 & 5 & 5 & 5\end{bmatrix}=\begin{bmatrix}0 & -1 & -4 & -4\\ 5 & 9 & 7 & 5\end{bmatrix}$$

12. Write 3 general translation matrices *T* so that the first produces only a *horizontal shift to the right*, the second a *vertical shift down*, and the third *a shift to the left and up*. For each T, determine *a* and b.

Sample answers:

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Horizontal shift right: $T=\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}$, a= 1 and b = 0 (a>0, b=0).

Vertical shift down : $T=\begin{bmatrix}0 & 0 & 0 & 0\\ -2 & -2 & -2 & -2\end{bmatrix}$, a= 0 and b = -2 (a=0, b<0).

Shift left and up: $T=\begin{bmatrix}-2 & -2 & -2 & -2\\ 2 & 2 & 2 & 2\end{bmatrix}$, a= -2 and b = 2 (a<0, b>0).

13. Write a general equation that we could use to find *M′* when given *M* and *T*.

$M+T=M'$

14. Set a = -2 and b = -3 and write an equation that shows this translation below. State the direction of the shift.

$$\begin{bmatrix}0 & -1 & -4 & -4\\ 0 & 4 & 2 & 0\end{bmatrix}+\begin{bmatrix}-2 & -2 & -2 & -2\\ -3 & -3 & -3 & -3\end{bmatrix}=\begin{bmatrix}-2 & -3 & -6 & -6\\ -3 & 1 & -1 & -3\end{bmatrix}$$

The shift is to the left 2 and down 3.