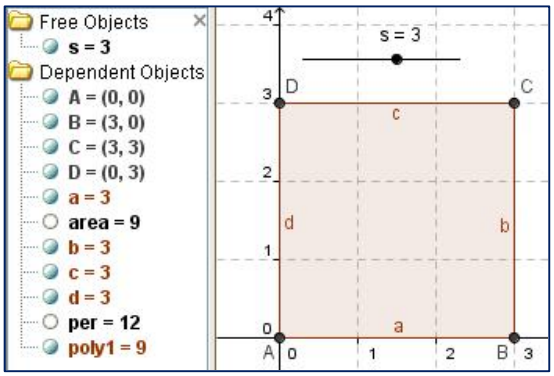
Constructing Squares with a Variable Slider in the Coordinate Plane



Goal: An introduction to creating and using a variable in a well-known setting – the square.

We will construct a slider whose current value will be the length of the side of the square. As we slide the slider to a new value, the side and size of the square will automatically change (vary).
That is, the slider is a variable.
We will then use this variable to make a formula for both perimeter and area.

Standard: California: CA4AF1.1 and CA4 MG2.0

Before: Teaching Activity: Constructing Squares in the Coordinate Plane

Note: A great feature of GeoGebra is that "the value" of a line segment is its length. It "looks" like a number and this value CAN be used as a number in a formula. But it CANNOT be made into a slider variable. Only a number can be used as slider and a slider variable is always just a number.

Setup GeoGebra for Construction (same as for Constructing Squares in the CP)

1. Open GeoGebra.
2. If not on, turn on the Algebra view and if on, turn off the Spreadsheet view.
   - To toggle, select commands: View -> Algebra view and View -> Spreadsheet view
3. Turn on the *Grid.*
   - Right-click in empty space in *Drawing pad* and select *Grid* from drop-down menu.
4. Move *Drawing pad* to show only first quadrant
   - Select *Move drawing pad* tool and then somewhere in the first quadrant, click and drag Drawing pad down and left so that (0,0) is in bottom left corner.
5. Use your mouse scroll button to zoom in/zoom out to get the view you want[1].
6. Turn on *Point capturing.*
   - GeoGebra usually does a good job of point capturing on Automatic. If you want to ensure that new points are on the grid, then select command:
     Options -> Point capturing -> On (Grid).
7. If you want, save these setting to default.
   - To have this be your default view when opening GeoGebra select command:
     Options -> Save Settings.

Construction: Draw a square whose side varies by moving a slider

1. Say: We want to create a slider whose values can be the side of a square. So the values must be positive numbers. Let's make a slider whose values are 0 to 5. Do: Select the Slider tool.
2. Say: We click anywhere in the *Drawing pad* to start the creation. Do: Click in the *Drawing pad* towards the top right corner. Say: Here is our dialog box where we tell it what we want.
   - We will name our slider *s* for "side". Type s (it will overwrite a).
   - We want the minimum value to be 0. Tab to or click in *Min* and type 0.

[1] To force the grid to be 1-1 and other options, see: Grid options

- We are done[2]. Click on Apply.

Immediately: Select the *Move* tool and say: all changes to the slider use the Move tool.

(If you forget and click and it wants to create another slider, just press the Esc key or click on the *Undo* button in the top right of the GeoGebra window. Been there, done that a zillion times.)

With the *Move* tool you can click and drag the slider line to a better position and if you don't see the label, right-click on the slider and select "Show label" from the drop-down menu.

3. Say: Let's make s=3 so our first square will have sides of length 3. Do: Select the *Move* tool and then click on the slider point and drag it to 3.
4. Say: Now we are ready to create our square. Let's put our first point, that is our bottom left corner point at (0,0). How did we do that before? We need the New point tool. Where do we click? Do: Select the *New Point* tool and click at (0,0). Note: Most probably GeoGebra is set to label this point with its name – in this case A. As before, you need to point out the definition of A=(0,0) in the Algebra view and connect it to the point in the Drawing pad.
5. Say: Let's think for a minute. For a square with side 3, what should be the coordinates of bottom right vertex of the square? We would count to the right 3 units on the horizontal and count nothing vertically. Since "H is before V" the coordinates would be (3,0).

   Say: But we want that "3" to change when we slide our slider. What is the name of the slider? s. So instead of (3,0), we want (s,0).

   Say: But we cannot click on the point (s,0). We must *input this formula*. How?

   Say: Down here in the Input bar. Let's click in the field. Do: Point to the Input bar at the bottom left of the GeoGebra screen and click. Say: Now our cursor is down here. Let's type in our formula. Do: Type: (s,0) and say and do: Hit Enter.

   Very important: Make sure B is a dependent object (see Figure 1)! Sometimes, we will accidently define B incorrectly and sometimes GeoGebra has a wild hair and will define it as a free object by actually substituting the current value of s. If you see that B is a free object (see Figure 2), double-click on it in the Algebra view. The definition will open and you should type s in place of 3 and hit Enter (see Figure 3). It should move to the dependent objects.
6. Say: Look – here is the new point B. It's coordinates are (3,0). Do: Point to the point in the *Drawing pad* and in the *Algebra view*. Say: Where is s? Let's run our mouse over B. See – it says Point B: (s,0). Do: Run mouse over B until definition block appears (see Figure 1).

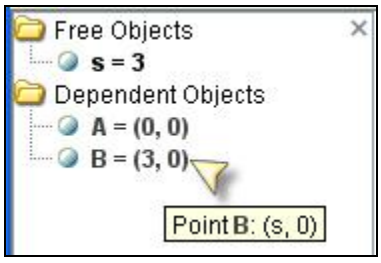


Figure 1: Correct

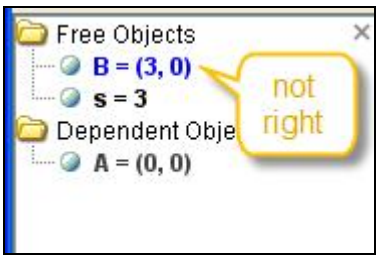


Figure 2: B should not be free

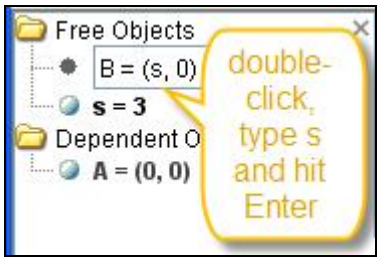


Figure 3: Fixing B

7. Say: Let's see if our slider variable works by changing our slider. Do: Select the *Move* tool and click and drag the slider point left and right. See: Point B should move left and right. Return the slider to s=3.

[2] You can make a minimum of 1 but kiddies like to see the square disappear ☺ and we will animate.

8. Say: Now what should be the coordinates of the top right vertex of our square. Do: Count up 3 from B and see (3,3). Say: We want the 3's to change with our slider so we want: (s,s). Let's type this formula for our point in the *Input bar*. Do: Type: (s,s) and say and do: Hit Enter.
9. Say: Here is our point C. We can see that it is at (3,3). Let's check the definition. Do: Run your mouse over C in the Algebra view until the definition block shows Point C: (s,s).
10. Say: Let's check that this point works with the slider. Do: Again, using the *Move* tool, click and drag the slider point s back and forth. See: Both B and C should move – B horizontally and C diagonally. Return the slider to s=3.
11. Say: Now what should be the coordinates of the top left vertex of our square. Do: Count up 3 from A=(0,0) and see (0,3). Say: We want the 3 to change with our slider so we want: (0,s). Let's type this formula for our point in the *Input bar*. Do: Type: (0,s) and say and do: Hit Enter.
12. Say: Here is our point d. We can see that it is at (0,3). Let's check the definition. Do: Run your mouse over D in the Algebra view until the definition block shows Point D: (0,s).
13. Say: Let's connect the points to see if we have a square. Do: Click on the *Polygon* tool and then on A, on B, on C and on D and again on A. Say: A perfect square!
14. Say: Let's see if our slider variable works. No matter what s is, we should always have a square. Do: Using the *Move* tool, click and drag the slider point s back and forth. The whole square should get larger and smaller but always be a square!

Using the *Move* tool, we suggest you move the labels for the points outside the square and the labels for the sides inside the square.

What can we do in GeoGebra and some Good Questions

1. Let's change this square to have sides of length 5. How do we do this? What will be the coordinates of B, C and D? Have the students write down the new coordinates and then use the *Move* tool to click and drag the slider to s=5.
2. We can make other formulas using s.
   a. For example, what is the formula for the perimeter of the square using s? So in the *Input bar*, we can type: per=4*s and hit Enter. With s=5, per=20. Use the *Move* tool to click and drag the slider to s=2 and watch the formula change to per=8. Roll your mouse over per in the Algebra view until the definition block shows Number per: 4 s.
   [Remember (a) although we can and did enter multiplication with a *, GeoGebra immediately replaced it with the (LaTeX) standard of a blank space. Since we do use a dot for multiplication, I usually think of this blank as a "microdot" so small I can't see it and (b) because the perimeter is a number, we keep with the GeoGebra standard of using small letters and not "Per" which would mean it was a point.]
   b. What is the formula for the area of a square using s? So in the *Input bar*, we can type: area=s*s and hit Enter. Remember that the value of poly1 is also the area of the square. So check that the value of area equals the value of poly1. Use the *Move* tool to click and drag the slider to s=1 and watch the formula change to area=1. Roll your mouse over area in the Algebra view until the definition block shows Number area: s s.
   [Remember - although the value of *area* and *poly1* are the same, we can always run our mouse over each of them and they will show their different definitions.]
3. We can animate the changing square. Right-click on the slider and select *Animation on* from the drop-down menu. Point out that the animation is a little jerky. (After setting *Animation on*, you can stop and start the animation using the Play and Pause buttons which are in the lower

left corner of the *Drawing pad*. When paused, you can always manually slide the slider point with your mouse.)

Some Good Questions

4. Say: How could we make our variable s be only whole numbers? Right-click on the slider and choose Properties from the drop-down menu. On the Basics tab, change the increment to 1 (from 0.1). Use the  *Move* tool to move the slider point manually. Point out that the slider is still a line but by setting the increment to 1 we are only "looking at" whole number values for s. Click on play to animate the slider. Now the animation is "really jerky" and slow. Mention that although the variable s can be any positive number, we are limiting our view to whole numbers. (Maybe we are building a square out of blocks so hard that they cannot be broken. Then our squares have only whole numbers for sides.)

   This is important because when we make tables for functions, we often just use integer values and then when we draw functions we draw the whole line. This is confusing, but with the slider and animation this idea becomes much clearer.

5. If you want, have them make a table (or open the spreadsheet and make a table) with values:

s	0	1				
per						

6. Say: How could we make our animation smooth? Right-click on the slider and choose Properties. Now change the increment to 0.01 and say 1/100. Try the slider manually and with animation. Now the animation is "smooth".

   Ask them if they want to make a table for all of these values? Explain to them that we CAN make a table with all of these values, but it would be really boring. That is why we usually just make tables with whole number values so that we get the idea of the pattern, but that all those intermediate values exist.

7. You can talk about the number of steps the animation makes from one end of the slider to the other. First think about the number of steps from 0 to 5 with an increment of 1. (Important: The number of steps is 5 and not 6! You cannot count a step when you haven't moved.) Then have them calculate the number of steps for an increment of 0.1 and of 0.01.
8. You can talk about the speed of the animation. (You can change the speed on the Slider tab.) In GeoGebra, the speed is how long it takes from to move the point from one end of the slider to the other. With an increment of 0.01, there are 100 more steps than with an increment of 1 so now the time between steps is $1/100^{th}$ of the time between steps when the increment was 1.
9. Mention that to be "really smooth" like in motion pictures, the increment is even smaller. Change the increment back to 0.1 and talk about how many steps the animation makes from one end of the slider to the other. Have them compare the time between steps now with the time between steps when the increment was 0.01 and again when the increment was 1. Make sure they make their sentences complete and correctly formulated.

Where to go from here?

1. Constructing rectangles in the coordinate plane.
2. Constructing rectangles with variables in the coordinate plane.
3. Constructing squares using perpendiculars.